Discrete and Computational Geometry, SS 18
Exercise Sheet "8": VC-dimension/Shatter function
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Thursday 27th of June**.
- *You may work in groups of at most two participants.*
- *You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.*

Exercise 22: Stabbing in 1D (4 Points)

Consider the problem of finding a minimal stabber (transversal) in dimension 1:

Given a finite set $\mathcal{R}$ of n intervals on the x-axis and a set $\mathcal{P}$ of m points on the x-axis, find a minimum subset $\mathcal{P}_{min} \subseteq \mathcal{P}$ such that each interval $\mathcal{I} \in \mathcal{R}$ contains at least one point of $\mathcal{P}_m in$ (i.e. $\forall \mathcal{I} \in \mathcal{R} : \mathcal{I} \cap \mathcal{P}_{min} \neq \emptyset$).

It was mentioned in the lecture that this can be solved efficiently with a sweep algorithm by adding a point every time the end of a not yet stabbed interval is reached.

Work out the details of this algorithm:

a) The content of the Sweep Status Structure (SSS)

b) The types of events in the Event Structure (ES)

c) The handling of an event (i.e. how does the SSS, ES and solution change)

d) Give the worst case running time and space requirements of your algorithm.

Exercise 23: Shatter Function Lemma (4 Points)

1. Show the correctness of

$$\binom{m-1}{i} + \binom{m-1}{i-1} = \binom{m}{i}.$$

2. Show that the bound (ii) in the Shatter Function Lemma is tight! Construct a set system $\mathcal{F}$ for all d and m such that $VCdim(\mathcal{F}) = d$ and $\pi_{\mathcal{F}}(m) = \Phi_d(m)$, where $\Phi_d(m) = \binom{m}{0} + \binom{m}{1} + \ldots + \binom{m}{d}$ holds.

3. Carify the proof detail on page 111 of the manuscript:

$$\left(1 - \frac{d}{m}\right)^{d-m}$$

is increasing in m!

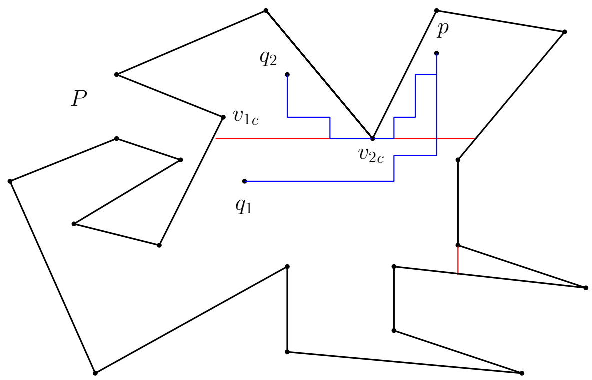

Figure 1: The points p and q_1 are L_1-visible whereas p and q_2 are not L_1-visible because the L_1-visibility is blocked by the horizontal L_1-cut of the locally Y-minimal vertex v_2.

Exercise 24: VC Dimension L_1-visibility (4 Points)

Consider the following notion of L_1-visibility inside a simple polygon P: Two points p and q inside P are L_1-*visible* to each other in P, iff there is an L_1-path inside P from p to q that is *monotone* in X- and Y-direction, see the Figure for some examples.

Try to find an example in order two show that the VC-Dimension of points in simple polygons is 3 (or even 4) w.r.t. L_1-visibility polygons of P!