

Discrete and Computational Geometry, SS 14
Exercise Sheet “2”: Chan’s technique/Lower bound
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until **Tuesday 29nd of April, 14:00 pm**. There will be a letterbox in the LBH building, close to Room E01.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at <https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom>*

Exercise 4: Constants in Chan’s method (4 Points)

We would like to find out how large the constants in the main lemma of Chan’s randomized technique might become. We refer to the proof of Lemma 4 and the application of computing the dilation of a chain.

For the *dilation-of-the-chain* computation we choose $\alpha = \frac{1}{2}$ and $r = 4$ for a decomposition as suggested.

Compute the constant R of Lemma 4 in the following way!

1. Assume that the decision algorithm runs in $C' \times n \log n$ time.
2. Choose an ϵ so that the precondition of Lemma 4 is fulfilled.
3. How many recursion steps l have to be done for your choice of ϵ ?
4. Express constant C in terms of precise values of l , α and r and the variable parameter C' .

Exercise 5: Lower bound: Computation of \hat{y} (4 Points)

This exercise is based on an inequality that was used in the proof for the lower bound on computing the graph-theoretic dilation of a given planar graph G . More precisely, the y coordinate of some points had to be chosen such that the inequality holds.

- a) Given constants n, y_ℓ, y^u , provide some value for y depending on n, y_ℓ, y^u that satisfies

$$\left(\frac{y - y_\ell + n + 0.5}{y - y^u + 1} \right)^2 < 1 + \frac{1}{n^2}$$

- b) Show that given $a, b > 0$ for each value $\epsilon > 0$ there exists some $\delta > 0$ such that the following holds:

$$1 - \epsilon < \frac{a + \delta}{b + \delta} < 1 + \epsilon$$

Exercise 6: Randomization arguments (4 Points)

Let us consider n different natural numbers. Let $q[1], \dots, q[n]$ denote a random permutation of these numbers. All permutations are chosen with the same probability.

The maximum of $q[1], \dots, q[n]$ can be computed by the following routine:

```
cntr = 0;
MaxSoFar = 0;
for  $j = 1$  to  $n$  do
  if (MaxSoFar <  $q[j]$ )
    then MaxSoFar =  $q[j]$ ;
    cntr = cntr + 1;
```

How often is *MaxSoFar* changed in the average? This means you have to give the expected value of the random variable *cntr* at the end of the routine.

Use the Θ -notation for your result.