Online Motion Planning, WT 13/14
Exercise sheet 3
University of Bonn, Inst. for Computer Science, Dpt. I

- *You can hand in your written solutions until Tuesday,* 12.11., 14:15*, in room E.06.*

Exercise 7: Exploring simple grid polygons (4 points)

In the construction for the lower bound on the competitive ration of any exploration strategy for simple grid polygons we have seen how to construct a grid polygon based on the moves of the algorithm, using a few gadgets; compare Figure 1.

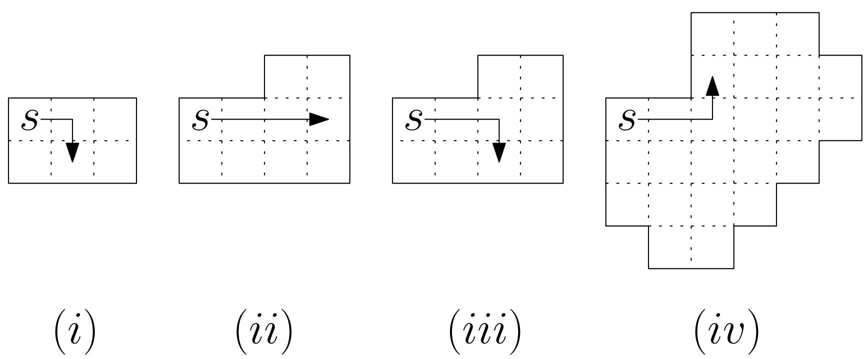

Figure 1: The gadgets used for the lower bound construction.

In this exercise, for simplicity, we focus on the case where only 2 gadgets are used – the starting gadget G_1 and *one* additional gadget, G_2. The gadgets are chosen depending on the moves of the exploration strategy, as before. The starting point is $s_1 \in G_1$, and we refer to the first cell of the second gadget which is explored by the algorithm as the starting point s_2 of the second gadget.

Prove that given any exploration tour T exploring the two gadgets – starting and returning to s_1 – where T that leaves and returns to G_1 exactly once, the following holds. If the first move of T in G_2 is to the right, then there exist two tours T_1 and T_2 starting in s_1 and s_2 respectively, where T_1 explores G_1 and T_2 explores G_2, such that

1. $|T| \geq |T_1| + |T_2| \geq \frac{7}{6}C(T)$ holds, if the cell where T leaves G_1 to G_2 is different than the cell where T returns to G_1 from G_2, and
2. $|T| \geq | T_1| + |T_2| + 1 \geq \frac{7}{6}C(T) + 1$ holds, if the cell where T leaves G_1 to G_2 is also the cell where T returns to G_1 from G_2.

Hint: For the construction of T_1 and T_2 use parts of tour T.

One assumption of the general construction is that the first step of the algorithm is to the right. However, this assumption cannot be maintained for the second gadget G_2. Suppose that the gadget G_1 is chosen as before. Now you are the adversary and choose a suitable existing gadget (do not create a new one) to be G_2 and show that if the first step of tour T (T is defined as above) is up or down, then there exists tours T_1 and T_2 as above, where $|T| \geq |T_1| + |T_2| + 1 \geq \frac{7}{6}C(T) + 1$ holds.

Exercise 8: Piecemeal and tethered reduction (4 points)

In the lecture it was shown that the piecemeal setting can be reduced to the tethered robot setting.

Formulate and prove the correctness of a reduction in the opposite direction. I. e. find a scheme that transforms a given piecemeal algorithm with $2(1+\alpha)r$ into a tethered-robot strategy with $(1 + \beta)r$ and figure out its cost factor, assuming $\alpha < \beta$.

Exercise 9: An example for the CFX algortithm (4 points)

Use the CFX algorithm to explore the graph G shown in Figure 2, starting in vertex s. Use the values $r = 4, \alpha = 1$ and $\ell = (1+\alpha)r = 8$.

Run the algorithm using the following assumptions.

- Any call of the subroutine $BoundedDFS(s, 8)$ will first start in the direction indicated by the arrow, i. e., visit the vertices $v_1, v_2, \ldots$ before vertex v_{10}.
- When constructing a spanning tree of a newly explored graph G', and G' contains edge (v_4, v_5), then the spanning tree of G' is constructed by removing edge (v_4, v_5) from G'.

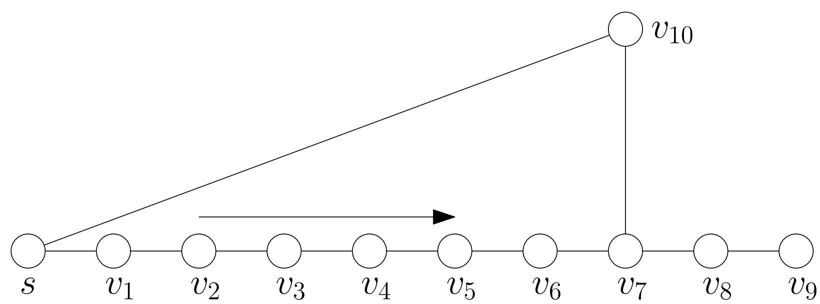

Figure 2: The "bad example" for *BoundedDFS*.