Online Motion Planning, WT 13/14
Exercise sheet 10
University of Bonn, Inst. for Computer Science, Dpt. I

- *You can hand in your written solutions until Tuesday,* 14.01., 14:15, *in room E.06.*

Exercise 28: The FIFO paging algorithm (4 points)

On a page fault, the *FIFO* algorithm evicts the page that has been in the cache longest.

1. Show that FIFO is *not* a marking algorithm.
2. Prove that FIFO is a conservative algorithm.

Exercise 29: The k-th harmonic number (4 points)

Prove that for any natural number $k \geq 1$

$$\ln k < H_k \leq 1 + \ln k,$$

where $H_k = \sum_{i=1}^{k} \frac{1}{i}$.

Exercise 30: The Full Access Cost Model (4 points)

We consider an alternate cost model the paging problem. We charge a cost of 1 for an access to the fast cache (serving a request whose page is already in the cache) and a cost of $s \geq 1$ for moving a page into the cache (additional cost for accessing the page are not included in s).

Let ALG be any marking algorithm. Given a request sequence σ, we denote by $ALG(\sigma)$ and $OPT(\sigma)$ the cost of ALG and the cost of an optimal offline algorithm, respectively for processing σ. Both OPT and ALG use a cache of size k. Let p denote the number of k-phases in σ, and $L(\sigma) = \frac{|\sigma|}{p}$ be the average length of a k-phase in σ. Show that

$$\frac{ALG(\sigma)}{OPT(\sigma)} \leq 1 + \frac{(k-1)s}{L(\sigma) + s}$$

holds for any σ. Furthermore conclude that this implies that ALG is $\frac{k(s+1)}{k+s}$-competitive. You may assume $L(\sigma) \geq k$.