Discrete and Computational Geometry, SS 18 Exercise Sheet "6": Brunn Minkowski inequality University of Bonn, Department of Computer Science I

- Written solutions have to be prepared until Thursday 14th of June.
- You may work in groups of at most two participants.
- You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.

**Exercise 16:** Concavity of volume functions (4 Points) Let  $A \subset \mathbb{R}^d$  be a set containing a single point and  $B \subset \mathbb{R}^d$  the unit hypercube.

- a) Give an explicit formula for the volume function v(t) = vol((1-t)A + tB).
- **b)** Prove  $v(t)^{\beta}$  is not concave on [0, 1] for any  $\beta > \frac{1}{d}$ .

## Exercise 17: Complexity of Minkowski-sum in $\mathbb{R}^2$ (4 Points)

Let P and Q be convex polygons with n and m edges respectively. Prove: The Minkowski-Sum  $P \oplus Q$  is a convex polygon with at most n + m edges!

## Exercise 18: Arithmetic and geometric mean (4 Points)

Show that for *n* reals  $x_1, x_2, \ldots, x_n$  the inequality

$$\frac{x_1 + x_2 + \dots + x_n}{n} \ge (x_1 \cdot x_2 \cdot \dots \cdot x_n)^{\frac{1}{n}}$$

holds.