# Online Motion Planning MA-INF 1314
# Restricted Graphexploration

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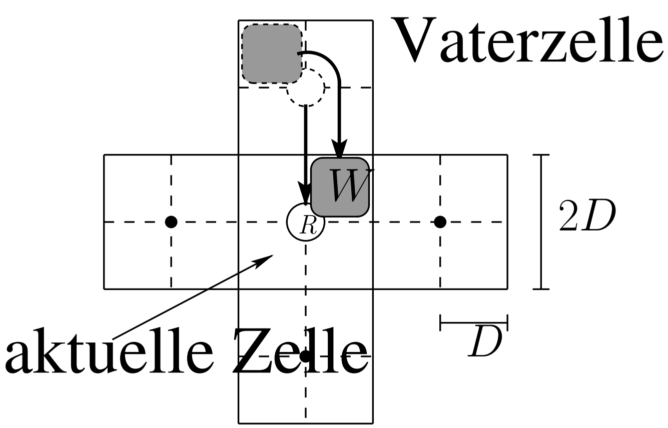
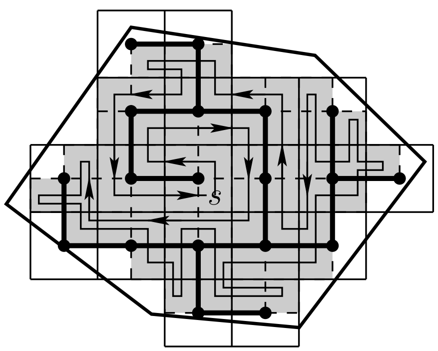
# Repetition!

- Modell 2D-cells, Spanning-Tree online construction
- SpiralSTC/ScanSTC: Detours along Spanning-Tree edge
- SpiralSTC equivalent to sub-cell-Modell!!
- Algorithmic formulation, recursively defined
- Strategy-Analysis: Locally!

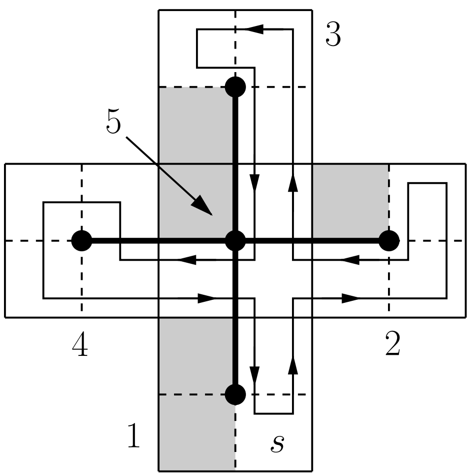



Belegt aber begehbare 2D Zellen

# Repetition: Local analysis

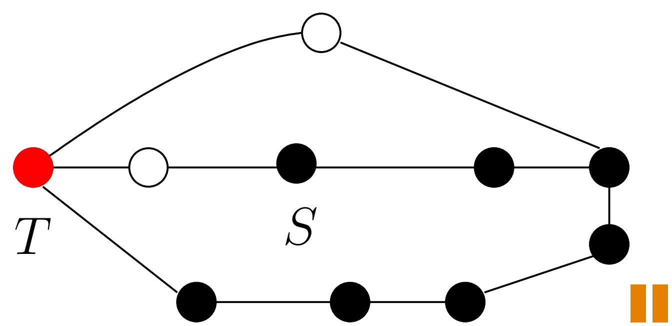
- Count the boundary cells
- Local analysis, multiple visits of cells, charge 2D cell
- Inner-cell (Responsibility)), Intra-cell
- Systematically: Boundary $D$-cells $\geq$ inner+intra




| Zelle | Übergr. | Intern | Gesamt | Randzellen |
|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 2 |
| 2 | 1 | 2 | 3 | 3 |
| 3 | 1 | 2 | 3 | 3 |
| 4 | 1 | 1 | 2 | 2 |
| 5 | 1 | 2 | 3 | 3 |

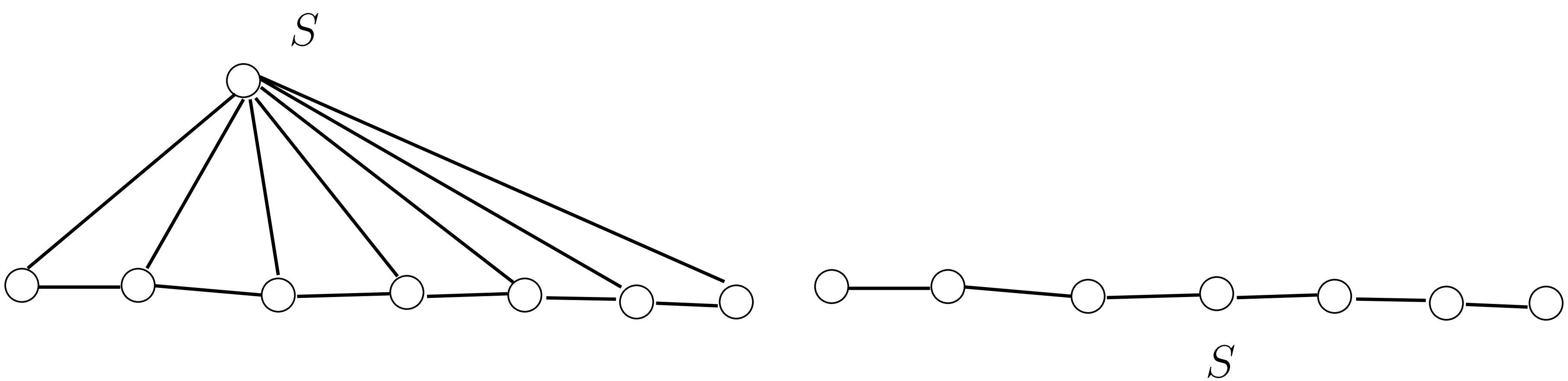
# Online graphexploration!

- Graph $G$: Visit all edges and vertices
- DFS $2$ competitive, optimal
- Searching $\Rightarrow$ Not too much into the depth
- Restricted exploration, tether/accum. (applications)



# Restricted online graphexploration

- Tether of length $k$
- Graph $G$: Depth $k$, longest shortest path to start
- Pure DFS: $k = 1$ but tether length $n$ is required
- BFS: $k \approx n/2$ but $\Omega(n^2)$ visits for $n$ edges



# Modell: Restricted (online) graphenexploration

1. Tethered agent $l = (1+\alpha)r$ (cable).
2. Agent returns to start after $2(1+\alpha)r$ steps (recharge accumulator)
3. Large graph, explore up to depth $d$, flexible $d$

- All vertices $r$ steps away, depth $r$ (radius)
- All edges length 1 (weights, exercise)
- Small look-ahead $\alpha$ necessary
- First variant, reduction for the others (Lemma/Exercise)

# Restricted graphexploration: Simulation

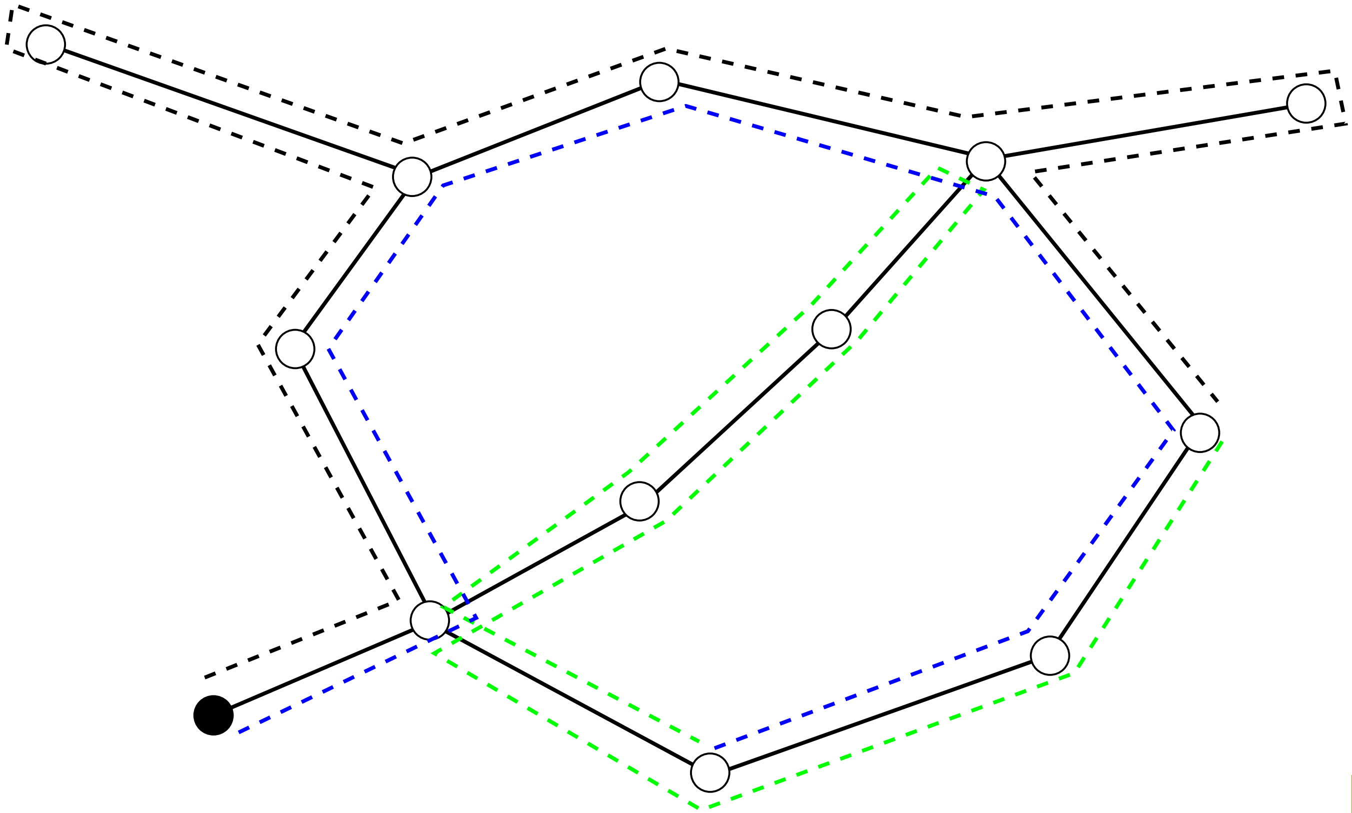
**Lemma** For any $\beta > \alpha$ a solution for the accumulation-variant with accumulator size $2(1+\beta)r$ can be attained from the solution of the tethered-variant with tether length $l = (1+\alpha)r$. The cost decrease by a factor of $\frac{1+\beta}{\beta-\alpha}$.

Proof: Blackboard!!!

# Offline Algorithmus: Accumulator-variant

- Offline: Graph is fully known
- Assume: $4r$ Accumulator
- Complexity, (NP-hard ?) unknown! Approximation $O(|E|)$!
- Algorithm: DFS $2|E|$ steps
- Cut into pieces of length $2r$, subpaths
- Starting segment in distance $r$
- Visit from start, explore subpath, move back!

# Example offline!



$$\left\lceil \frac{2|E|}{2r} \right\rceil \times 2r + 2|E| \le 6|E|$$ Example: $r = 5$

# Offline Algorithm: Accumulator-variant

**Lemma** A simple Accumulator-Offline Algorithm visits at most $6|E|$ edges.

- Reach any subpath-start with step-length $2r$
- Explore all subpath: $2|E|$
- $\left\lceil \frac{2|E|}{2r} \right\rceil$ subpaths in total
- Reaching by $\left\lceil \frac{|E|}{r} \right\rceil 2r$ steps
- $\left\lceil \frac{|E|}{r} \right\rceil 2r \leq \left( \frac{|E|}{r} + 1 \right) 2r \leq 2|E| + 2r$
- $4|E| + 2r \leq 6|E|$

# Online: Tethered graphexploration

- Tether variant (cable), reductions for others (Lemma/Exercise)
- First idea, DFS (edges) until tether is fully used, then backtracking
- bDFS, bounded DFS
- Nice try, is not enough!

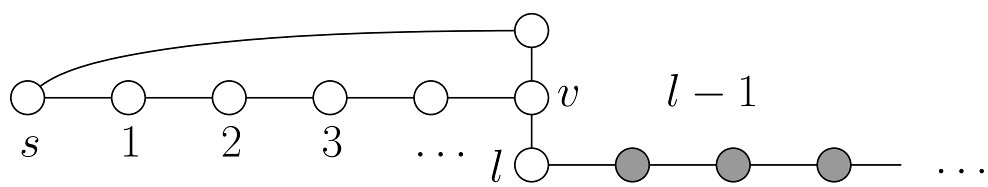
# Method: Bounded DFS

**bDFS(** $v$**,** $l$ **)**:

- **if** $(l = 0) \vee$ (all outgoing edges are explored) **then**
  RETURN
  **end if**
  **for all** non-explored edge $(v, w) \in E$ **do**
  Move from $v$ to $w$ by $(v, w)$.
  Mark $(v, w)$ as *explored*
  bDFS($w$, $l - 1$).
  Move back from $w$ to $v$ by $(v, w)$.
  **end for**

# Bounded DFS

- Example unit-length edge
- Problem: Not all edges will be reached
- Edge to $v$ is marked, End!
- Only bDFS is not enough



# CFS Algorithm: Mark the vertices

**non-explored** vertices, never visited.

**incomplete** visited vertices, but there are non-explored edges starting at $v$.

**explored** vertices, all incident edges have been explored.

# CFS Algorithm

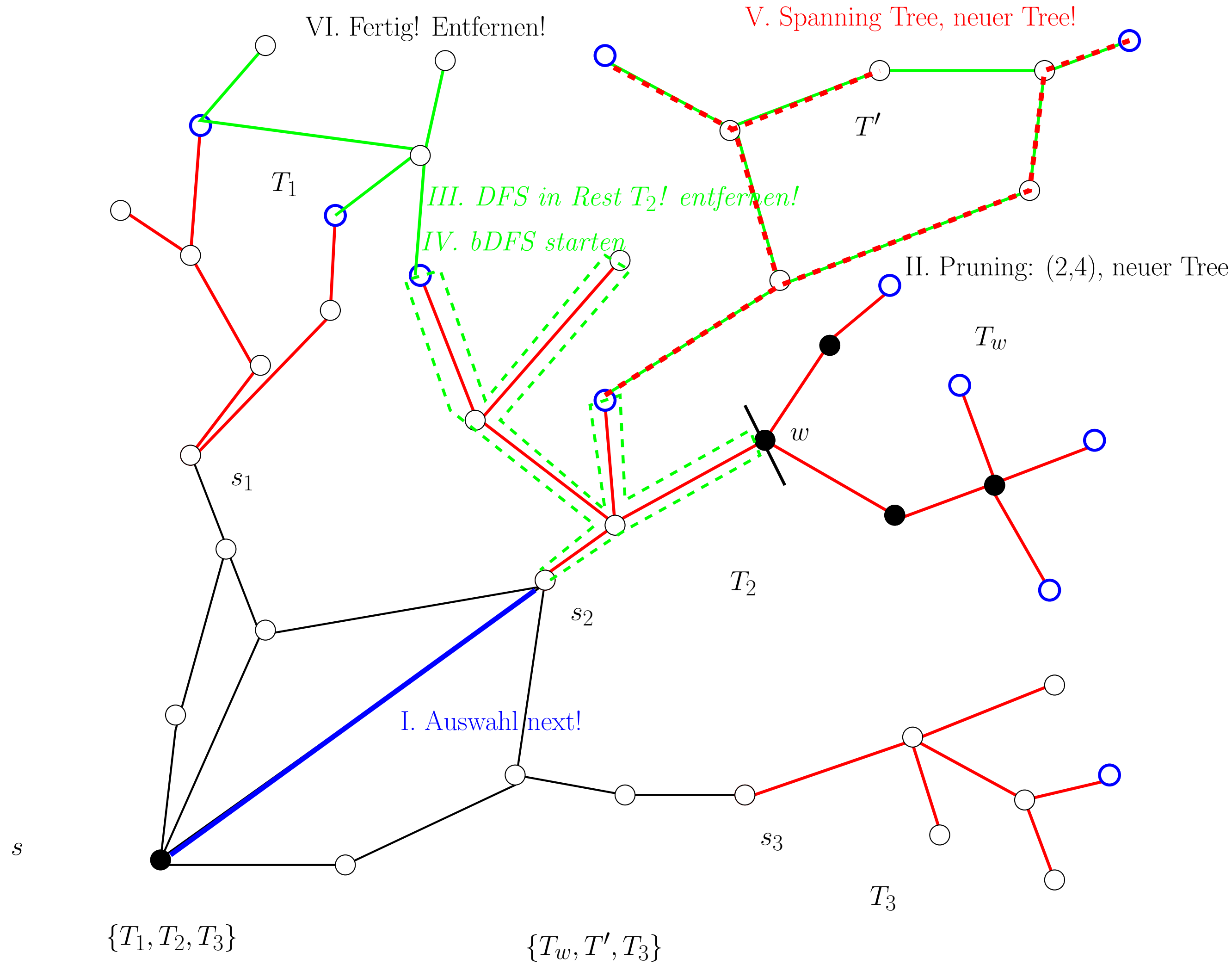
- Start bDFS at different sources
- Set of (edge) disjoint **trees** $\mathcal{T} = \{ T_1, T_2, \ldots, T_k \}$
- Root vertices $s_1, s_2, \ldots, s_k$
- Choose $T_i$ with $s_i$ closest to $s$, move to $s_i$
- Pruning of $T_i$: Build $T_{w_j}$ with root $w_j$ if:

1. $d_{T_i}(s_i, w_j) \geq \mathit{minDist} = \frac{\alpha r}{4}$
2. $\mathit{Depth}(T_{w_j}) \geq \mathit{minDepth} - \mathit{minDist} = \frac{\alpha r}{4}$

- Add all $T_{w_j}$ to $\mathcal{T}$! Remove $T_i$ from $\mathcal{T}$
- Explore $T_i$ without $T_{w_j}$ from $s_i$ by DFS and
- start bDFS at the incomplete vertices

- Graph $G'$ of new vertices and edges
- Build a spanning tree $T'$ of $G$
- Choose root $s'$ with minimal distance to $s$
- Add all these trees to $\mathcal{T}$
- Special case: Trees in $\mathcal{T}$ gets fully explored
- Trees in $\mathcal{T}$ with common egdes are joined
- Merging: Build spanning tree with new root

# CFS Algorithm, Example



# CFS Algorithm

## CFS( $s$, $r$, $\alpha$ )

- $\mathcal{T} := \{\{s\}\}$.
  **repeat**
  $T_i :=$ tree in $G^*$ closest to $s$.
  $s_i :=$ root of $T_i$ (closest vertex to $s$).
  $(T_i, \mathcal{T}_i) :=$ prune( $T_i$, $s_i$, $\frac{\alpha r}{4}$, $\frac{\alpha r}{2}$ ).
  $\mathcal{T} := \mathcal{T} \setminus \{T_i\} \cup \mathcal{T}_i$.
  explore( $\mathcal{T}$, $T_i$, $s_i$, $(1+\alpha)r$ ).
  Remove all fully explored trees from $\mathcal{T}$.
  Merge all trees in $\mathcal{T}$ with common vertices.
  Calculate spanning tree/root for merged trees.
  **until** $\mathcal{T} = \emptyset$

# CFS Algorithmus: Pruning!

**prune(** $T$, $v$, $minDist$, $minDepth$ **)**

- $v :=$ Root of $T$.
  **for all** $w \in T$ such that $d_T(v, w) = minDist$ **do**
  $T_w :=$ subtree of $T$ with root $w$.
  **if** max. distance from $v$ and vertex in $T_w > minDepth$ **then**
  // Cut-Off $T_w$ from $T$:
  $T := T \setminus T_w$.
  $\mathcal{T}_i := \mathcal{T}_i \cup \{T_w\}$.
  **end if**
  **end for**
  RETURN $(T, \mathcal{T}_i)$

# CFS Algorithmus: Explore!

**explore( $\mathcal{T}$, $\boldsymbol{T}$, $s_i$, $l$ )**

- Move from $s$ to $s_i$ along shortest (known) path.
  Explore $T$ by DFS. If incomplete vertex $v$ is visited:
  $l' :=$ remaining tether length.
  bDFS( $v$, $l'$ ).
  $E' :=$ newly explore edges.
  $V' :=$ vertices from in $E'$ (plus $v$).
  Build spanning tree $T'$ of $G' = (V', E')$.
  $\mathcal{T} := \mathcal{T} \cup \{T\}$.
  Move back from $s_i$ to $s$.

# CFS Algorithmus: Example!!

- $G^* = (V^*, E^*)$ Graph of the explored edges and and vertices (successively extended)
- Set $\mathcal{T}$
- Pruning
- Explore (DFS/bDFS)