

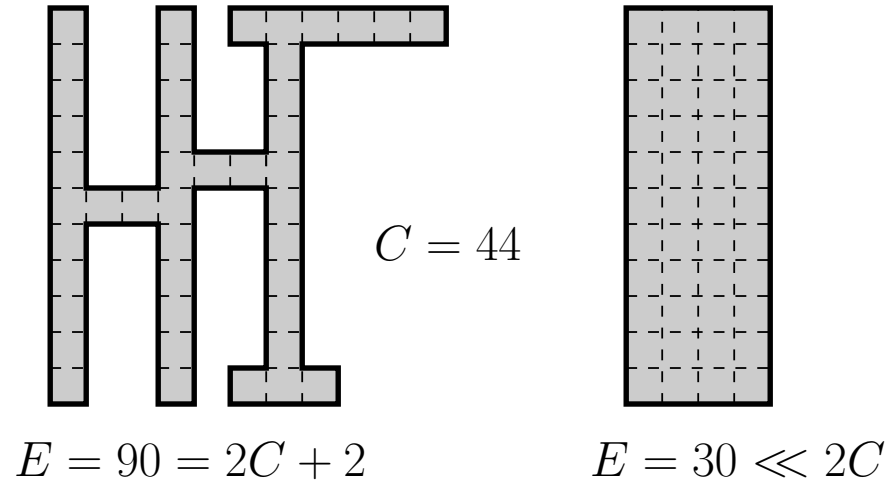
# Online Motion Planning MA-INF 1314

## Smart DFS

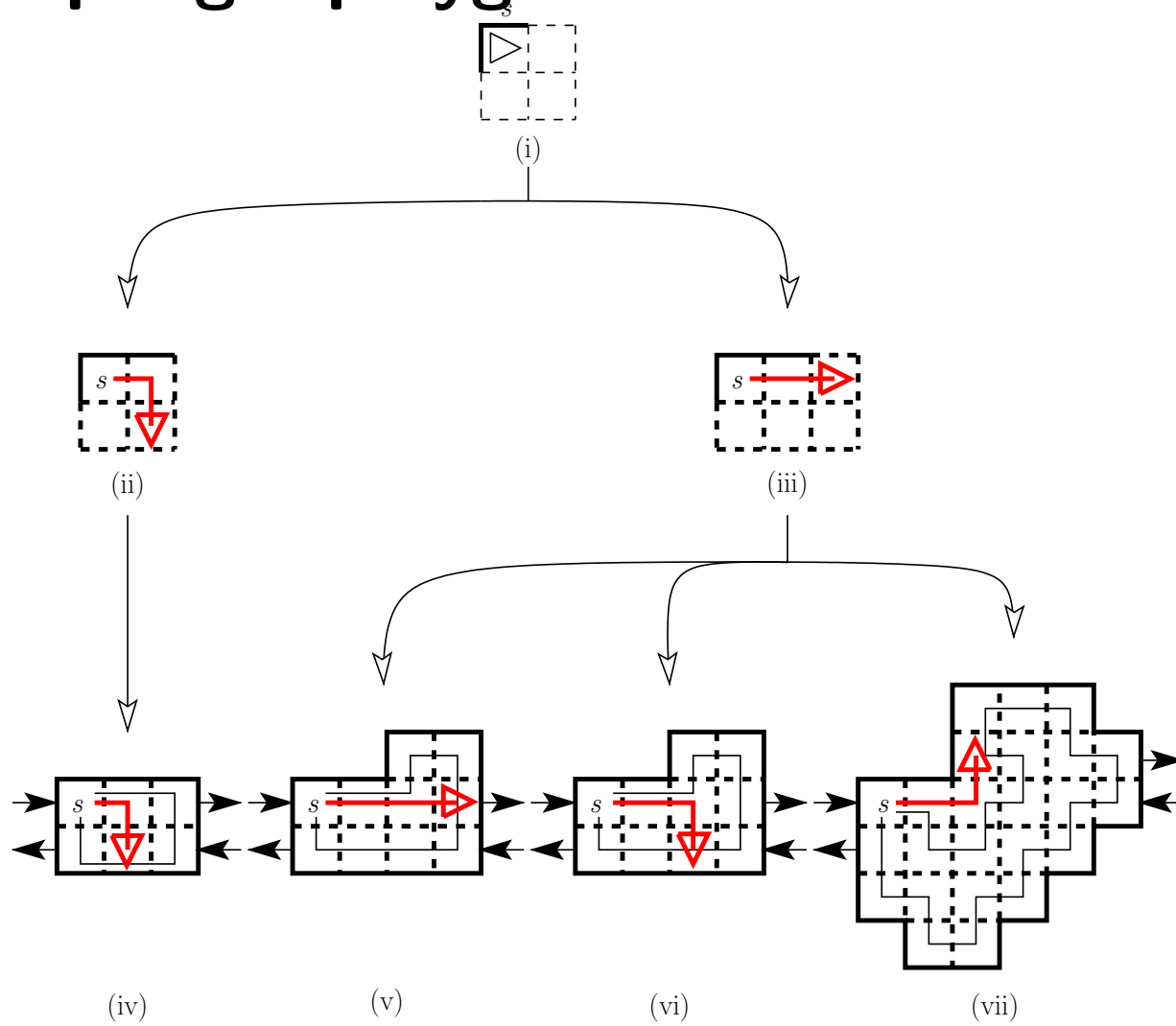
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# Repetition: Simple gridpolygons

- Pure DFS is not the best idea
- Relation between edges and cells

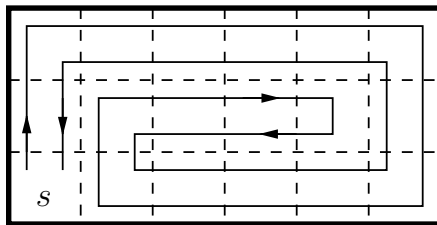


# Simple gridpolygons: Lower bound! $\frac{7}{6}!$

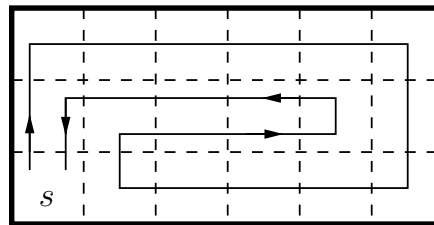


# Improve DFS

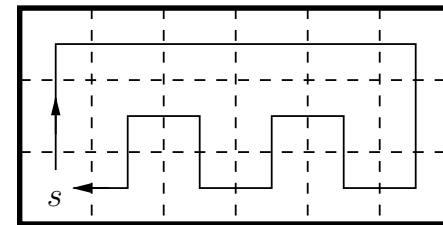
- Better than ratio 2 in fleshy environments
- Visit only the vertices
- **Smart DFS!**
- Number of steps:  $C + \frac{1}{2}E - 3$
- $\frac{4}{3}$  competitive



DFS



Verbesserung

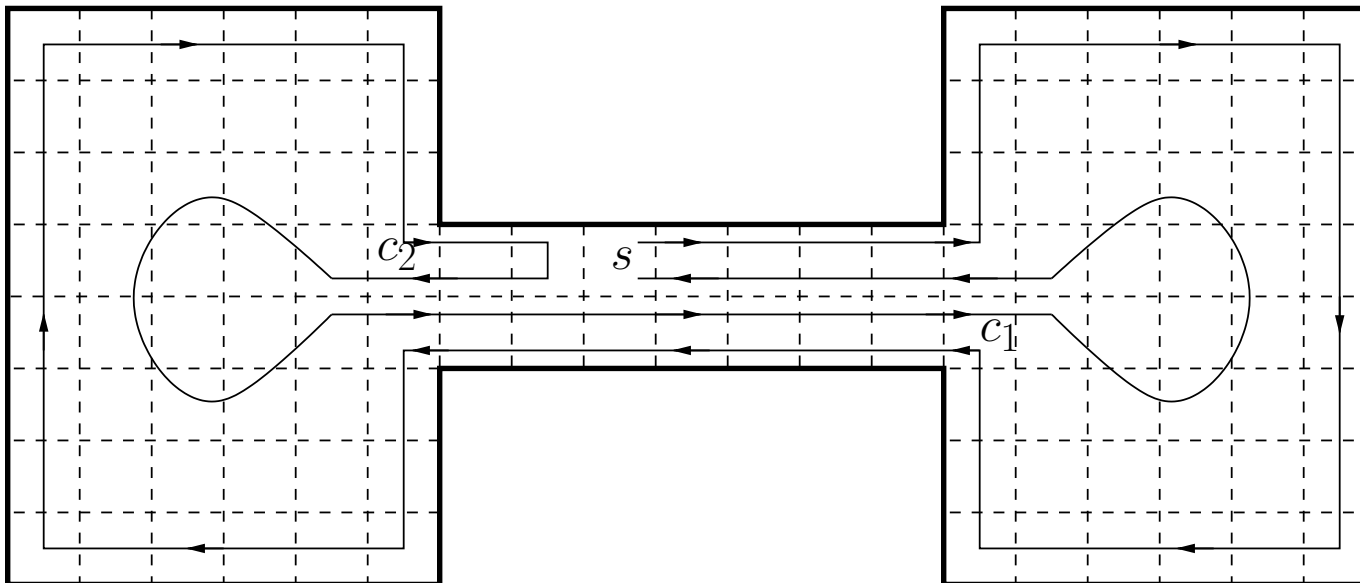


Optimal

# Two improvement for DFS

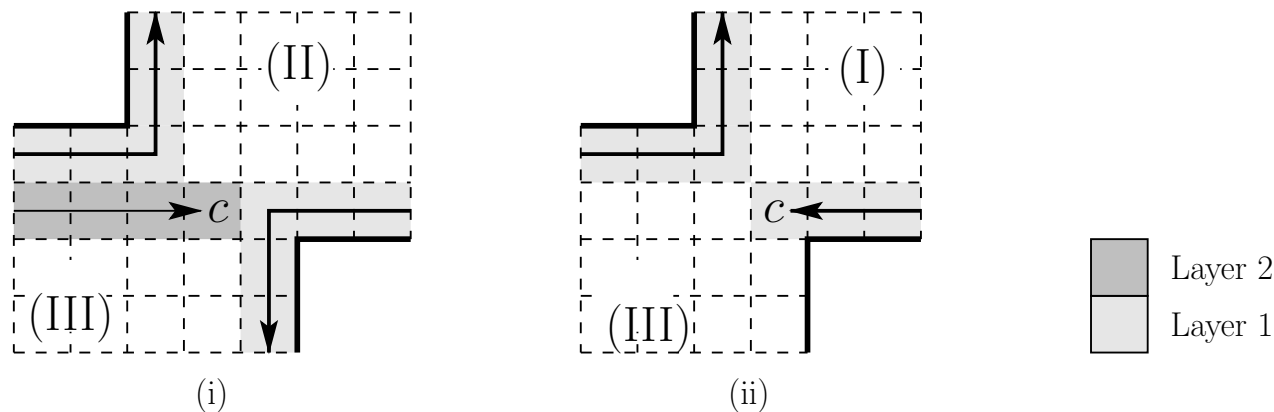
First idea: Move along the shortest path to the next *free* cell!

Second idea: Split into different areas happens: Work on the part where the starting point is not inside! Farther away!



# Smart DFS: 2. Improvement!

- **Split-cell** occurs in **Layer  $l$** : How to proceed?
  - Where is the starting point?
    - Component  $K_i$  *fully* enclosed by Layer  $l$ .
    - Component  $K_i$  *not* visited by Layer  $l$ .
    - Component  $K_i$  *partially* enclosed by Layer  $l$ .
      - Visit component of type (III) last! Starting point!
      - Special cases: One step Right-Hand-Rule



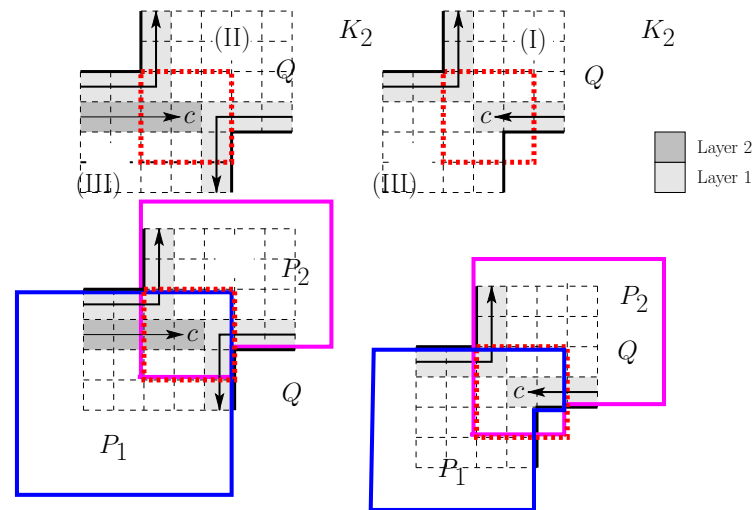
# Lemmata!

**Lemma:** The  $l$ -Offset of a simple gridpolygon  $P$  has *at least*  $8l$  edges less than  $P$ .

**Lemma:** The shortest path between to cells  $s$  and  $t$  of a simple gridpolygon  $P$  with  $E(P)$  edges has at most  $\frac{1}{2}E(P) - 2$  steps.

# Decomposition of $P$ at split-cell

- Decomposition Rectangle  $Q$ :  $2q + 1$  ■
- Cases:  $K_2$  of type I) ( $q = l$ ) or vom type II) ( $q = l - 1$ ) ■
- $P_2$ , such that  $K_2 \cup \{c\}$  is the  $q$ -Offset of  $P_2$  ■
- $P_1 := ((P \setminus P_2) \cup Q) \cap P$  ■ Intersection with  $P$  for the movements ■



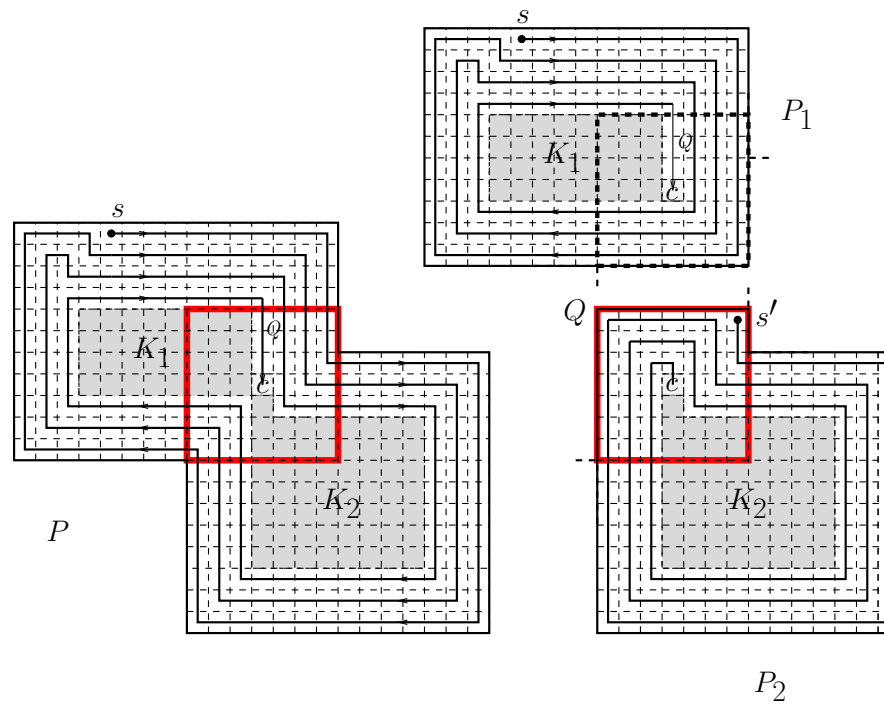


# Analysis: Visity beyond the cells

- Any cell is visited once■
- Number of steps  $S(P)$ : Visit cells (-1) plus additional visits■
- $S(P) := C(P) + excess(P)$ ■
- Calculate  $excess(P)$ ■

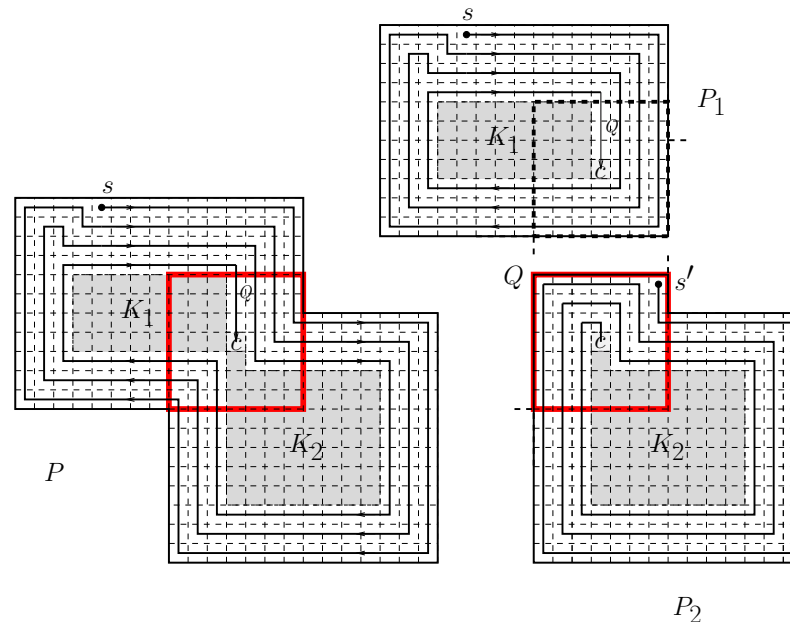
# Excesslemma

**Lemma:**  $P$  gridpolygon and  $c$  a split-cell, such that  $P$  splits into  $K_1$  and  $K_2$  (for the first time). Let  $K_2$  be the component, that is visited first. We have:  $\text{excess}(P) \leq \text{excess}(P_1) + \text{excess}(K_2 \cup \{c\}) + 1$ . ■



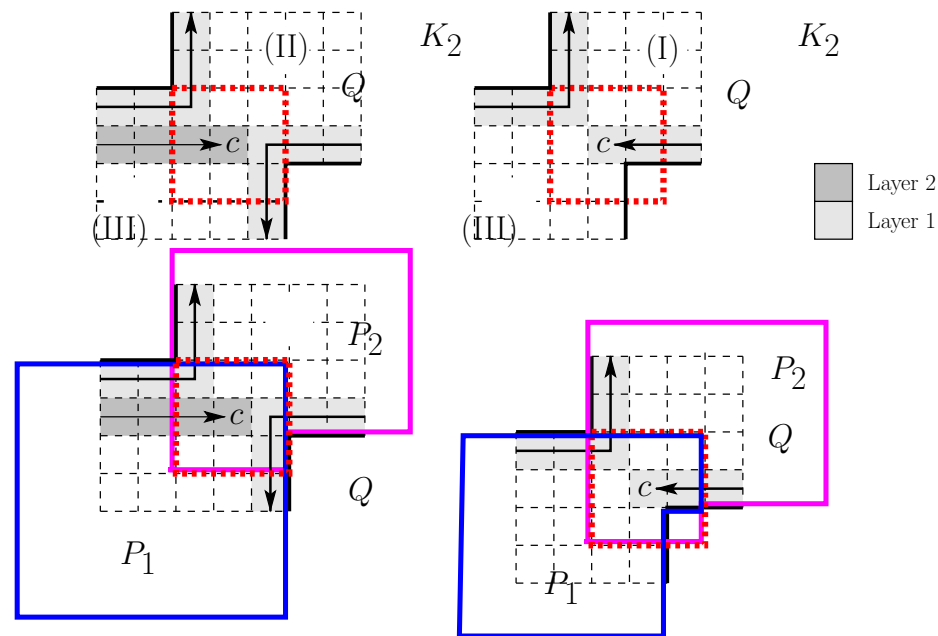
$$\text{excess}(P) \leq \text{excess}(P_1) + \text{excess}(K_2 \cup \{c\}) + 1.$$

- Explore  $K_2 \cup \{c\}$  after  $c$  by SmartDFS, return to  $c$  ■
- Gives: max.  $\text{excess}(K_2 \cup \{c\})$  since  $P_2 \setminus (K_2 \cup \{c\})$  optimal ■
- $c$  twice: plus 1 ■
- Then move to  $P_1$ : Maximal  $\text{excess}(P_1)$  ■



# Relationship edges of $P$ and $Q$ Lemma 1.14

**Lemma:**  $P, P_1, P_2$  und  $Q$  as given. For the number of edges we have  $E(P_1) + E(P_2) = E(P) + E(Q)$ . ■



## Relationship edges of $P$ und $Q$

$$E(P_1) + E(P_2) = E(P) + E(Q). \blacksquare$$

Two arbitrary gridpolygons  $P_1$  und  $P_2$  gilt:

$$E(P_1) + E(P_2) = E(P_1 \cup P_2) + E(P_1 \cap P_2). \blacksquare$$

For  $Q' := P_1 \cap P_2$ , we have:  $\blacksquare$

$$E(P_1) + E(P_2) = E(P_1 \cap P_2) + E(P_1 \cup P_2)$$

$$\blacksquare = E(Q') + E(P \cup Q)$$

$$\blacksquare = E(Q') + E(P) + E(Q) - E(P \cap Q)$$

$$\blacksquare = E(P) + E(Q), \text{ da } Q' = P \cap Q$$

$\blacksquare$

# Exploration Theorem

**Theorem:** SmartDFS explores a simple gridpolygon  $P$  with  $C$  cells and  $E$  boundary edges with at most  $C + \frac{1}{2}E - 3$  steps. ■

Proof: ■ Induction over number of components ■

- **Induction base:** One component ■
- Visit cells:  $C - 1$ , back to start ■
- Shortest path Lem:  $\frac{1}{2}E(P) - 2 + C - 1 = C + \frac{1}{2}E - 3$  ■

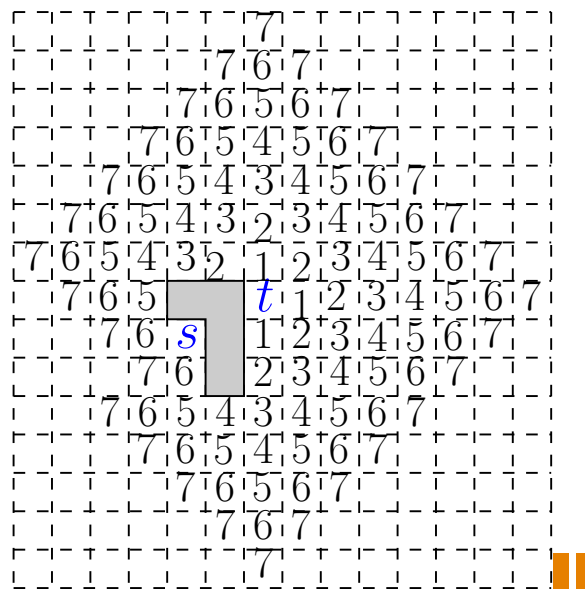


$$\mathbf{I} = \frac{1}{2}E(P) - 3\mathbf{I}$$



# Shortest path over explored cells

- Offline Problem, within SmartDFS
- Wave front from  $t$  to  $s$ , label with  $L_1$  distance to  $t$
- Mark adjacent cell with label + 1, Queue



# Algorithm of Lee: Labels

Datenstruktur: Queue  $Q$

{Initialise:}

■  $Q.InsertItem(t)$ ;

Markiere  $t$  mit 0;

{Wave Propagation:}

**loop**

$c := Q.RemoveItem()$ ;

**for all** Cells  $x$  such that  $x$  adjacent to  $c$  and  $x$  not marked **do**

        Mark  $x$  with label  $label(c) + 1$ ;

$Q.InsertItem(x)$ ;

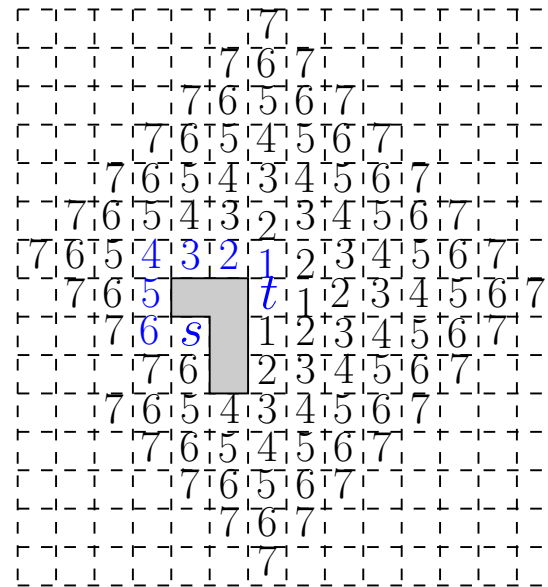
**if**  $x = s$  **then** break loop;

**end for**

**end loop**

# Algorithm of Lee: Compute the path

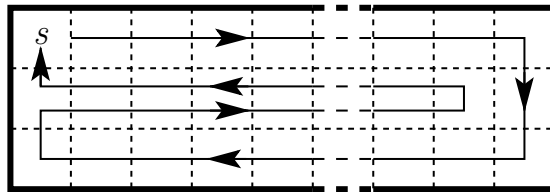
Move along cells with decreasing labels starting from  $s$  nach  $t$ .



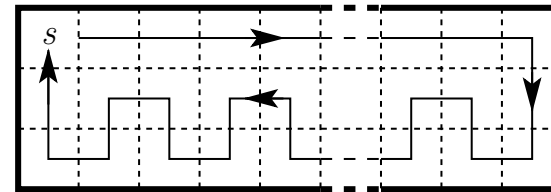
Not unique!

# Competitive Ratio: SmartDFS

- Compare optimal path vs. SmartDFS
- THIS is the worst-case
- $S(P) \leq \frac{4}{3} C(P) - 2$
- $3 \times m$ ,  $m$  even, exact! I.e., 30 against 24!



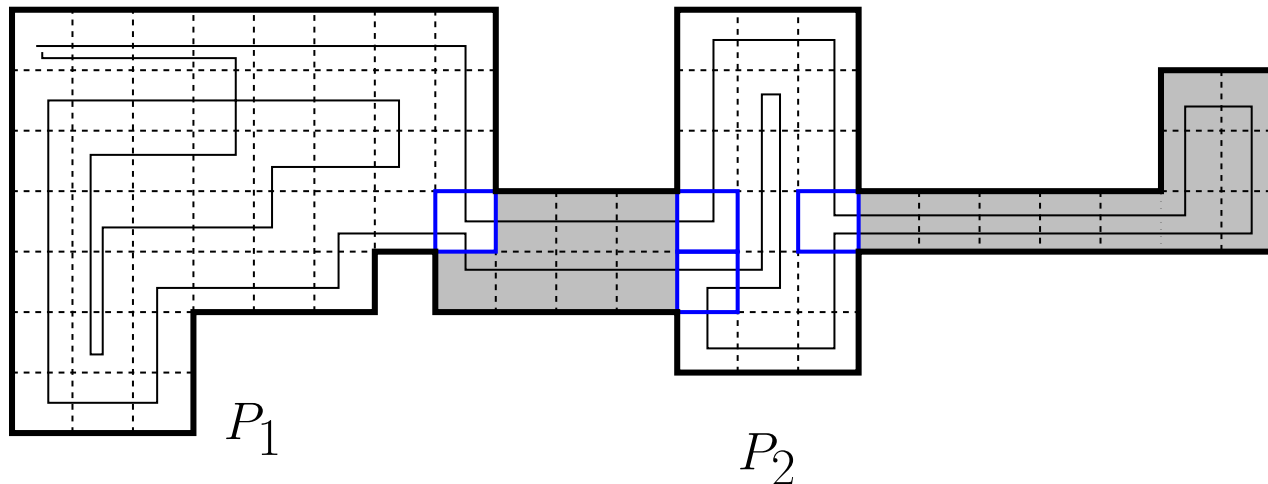
SmartDFS



Optimale Strategie

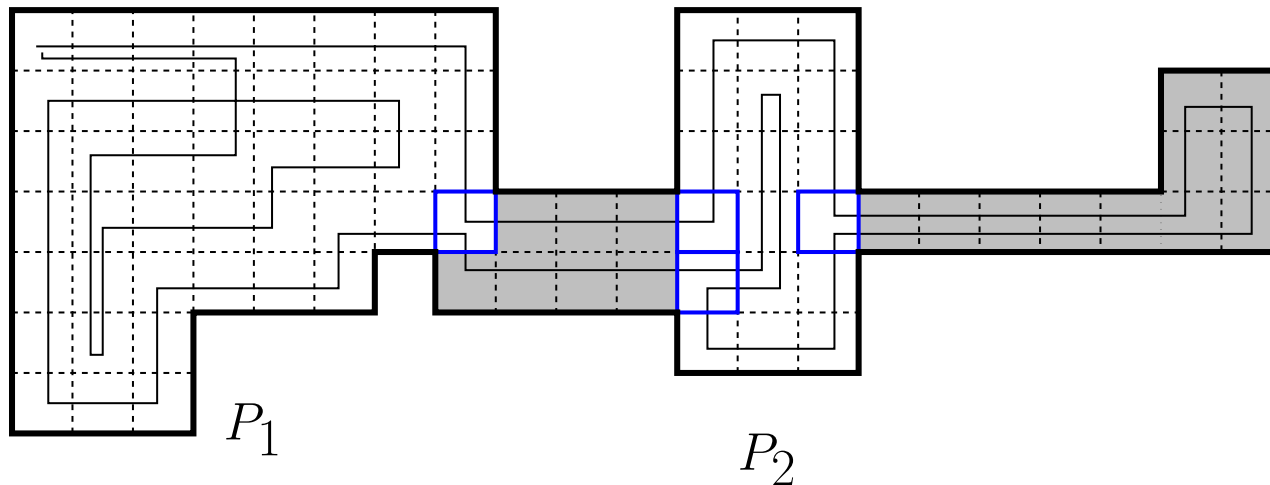
# Structural properties: SmartDFS

- Optimal in *corridors* of width 1/2
- Cells that do not change the layer-number of the neighbors, if we delete them!
- Neighbours of layer 1! Gates for the corridors!
- **Def.** Narrow passages!



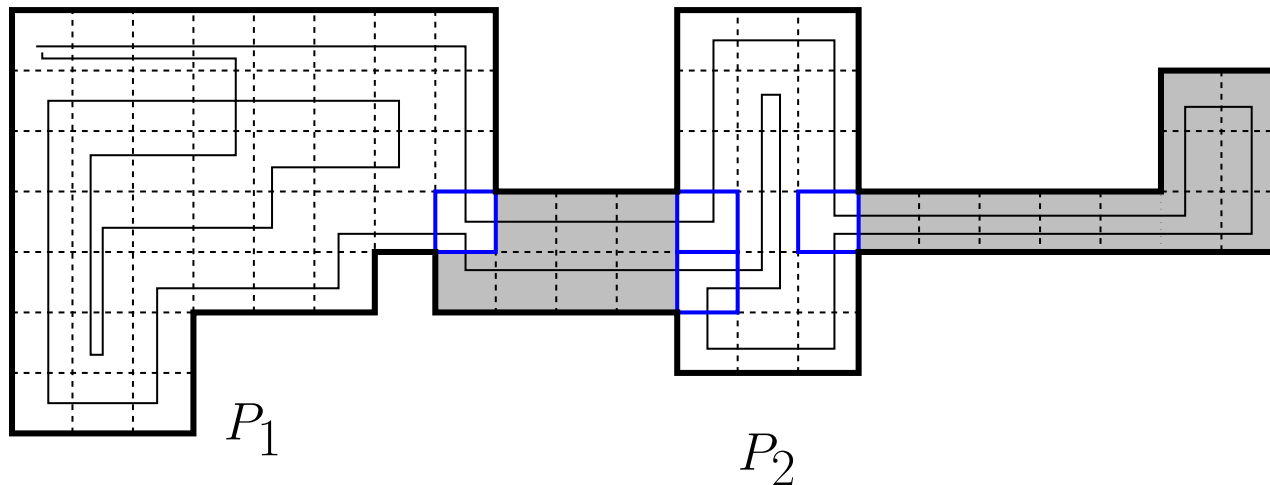
# Omit narrow passages

- Omit *narrow passages*, analyse the remaining parts
- Results in sequence of polygons  $P_i, i = 1, \dots, k$
- Example  $P_1$  und  $P_2$ : Analysis with the cells!!



# Omit narrow passages

- Analyse Polygone without *narrow passages*!
- Inductively over the split-cells in their first layer!
- Induction base: No split-cell in the first layer!
- Analyse such polygons first!



# Polygons without *narrow passages* and without 1-Layer split

**Lemma:**  $P$  simple gridpolygon, no narrow passages, no split in the first layer.  $E(P) \leq \frac{2}{3}C(P) + 6$  holds. **Beweis:**

- Exactly for  $3 \times 3$  gridpolygons
- 9 cells, 12 edges
- Reduce any such gridpolygon to this base case
- Subtraction of at least 3 cells and at most 2 edges
- Properties survive

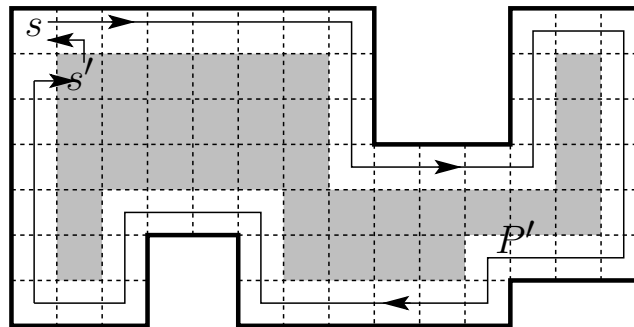




# Polygons without *narrow passages* and without 1-Layer split

**Lemma:** SmartDFS requires  $S(P) \leq C(P) + \frac{1}{2}E(P) - 5!$

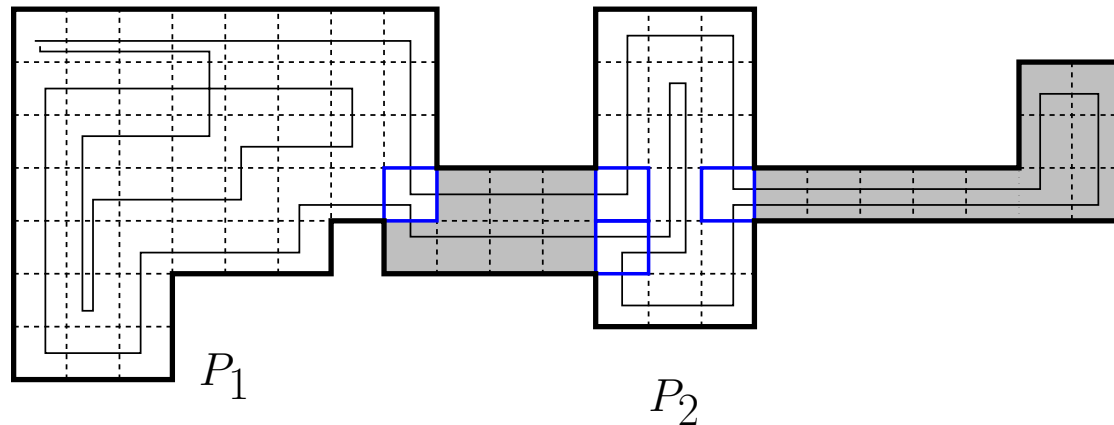
- Theorem:  $S(P) \leq C(P) + \frac{1}{2}E(P) - 3$
- By assumption: SmartDFS full first round,  $C'$  steps (1-Layer)
- SmartDFS starts at  $s'$  in  $P'$





# Theorem: SmartDFS ist $\frac{4}{3}$ kompetitiv

- Narrow passages optimal, sequence of  $P_i$  independently!
- Only cells and steps, no edges!!
- Induction in  $P_i$  over split-cell number!  $S(P_i) \leq \frac{4}{3}C(P_i) - 2$
- Induktion base: Use special lemmata!



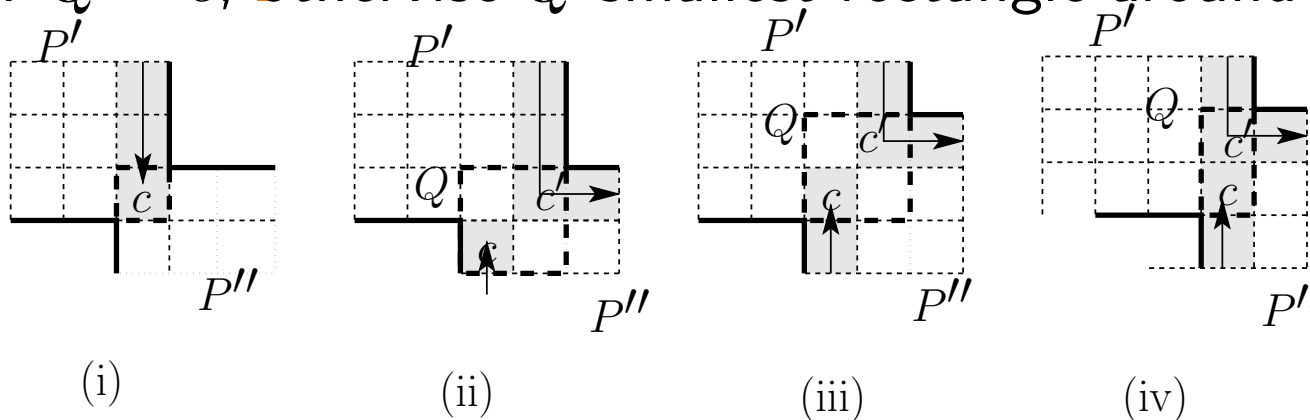
**Induction base:**  $S(P_i) \leq \frac{4}{3}C(P_i) - 2$

- $P_i$  no split-cell means, no split-cell in Layer 1
- Apply case-sensitive Lemma:  $C(P) + \frac{1}{2}E(P) - 5$
- Apply structural Lemma:  $E(P) \leq \frac{2}{3}C(P) + 6$

$$\begin{aligned} S(P_i) &\leq C(P_i) + \frac{1}{2}E(P_i) - 5 \\ &\leq C(P_i) + \frac{1}{2} \left( \frac{2}{3}C(P_i) + 6 \right) - 5 \\ &= \frac{4}{3}C(P_i) - 2 \end{aligned}$$

## Induktion step: $S(P_i) \leq \frac{4}{3}C(P_i) - 2$

- Split-cell in first layer of  $P_i$ , otherwise done: Two Cases
- Split by  $c$  adjacent to some  $c'$
- Typ (I) (curr. layer not) or Typ (II) (curr. layer fully.) component
- Split into  $P'$  and  $P''$  with Rectangle/Square  $Q$
- Case (i):  $Q = c$ , otherwise  $Q$  smallest rectangle around  $c, c'$

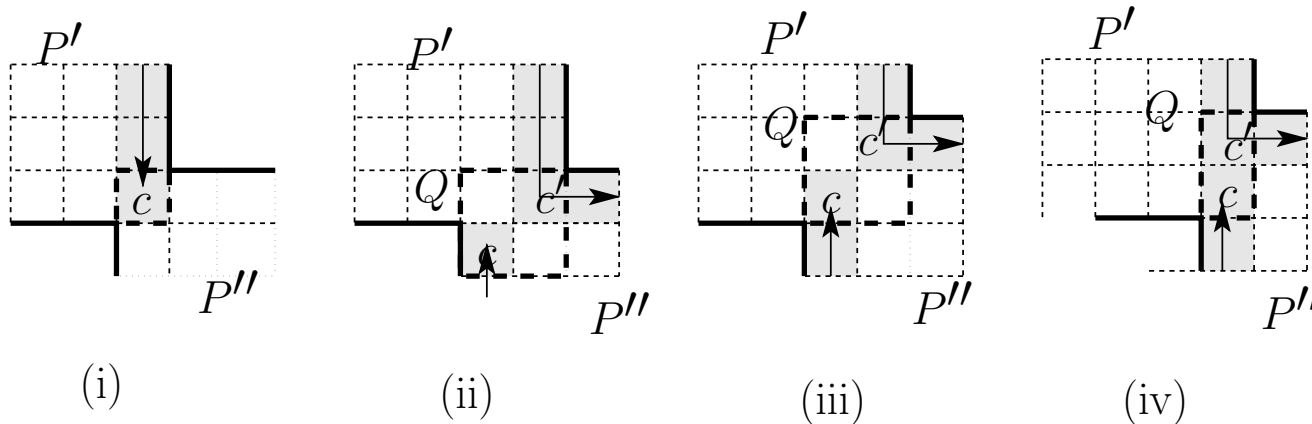


## Case (i): $S(P_i) \leq \frac{4}{3}C(P_i) - 2$

- $S(P_i) = S(P') + S(P'')$  (Gate)  $C(P_i) = C(P') + C(P'') - 1$
- Induction: For  $P'$  and  $P''$  (less split-cells)

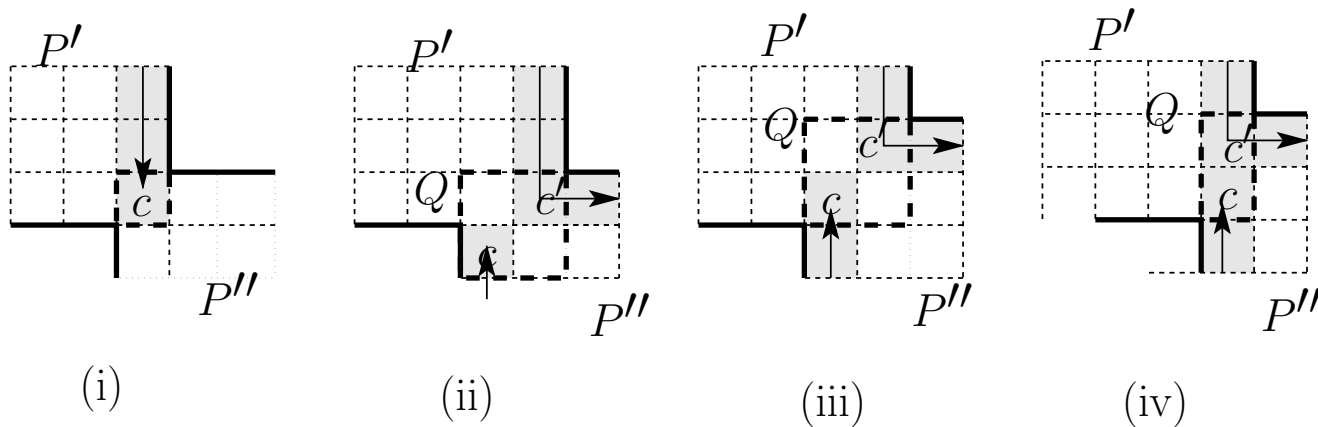
$$S(P_i) = S(P') + S(P'') \leq \frac{4}{3}C(P') - 2 + \frac{4}{3}C(P'') - 2$$

$$\leq \frac{4}{3}C(P_i) + \frac{4}{3} - 4 < \frac{4}{3}C(P_i) - 2$$



## Fall (ii),(iii): $S(P_i) \leq \frac{4}{3}C(P_i) - 2$

- $|Q| = 4$  but save 4 steps! ■
- $P', P''$  separately (I.A.) but ■
- Path in  $P_i$  from  $c'$  to  $c$  or from  $c$  to  $c'$  done in  $P', P''$  ■
- Save at least  $4=|Q|$  steps ■



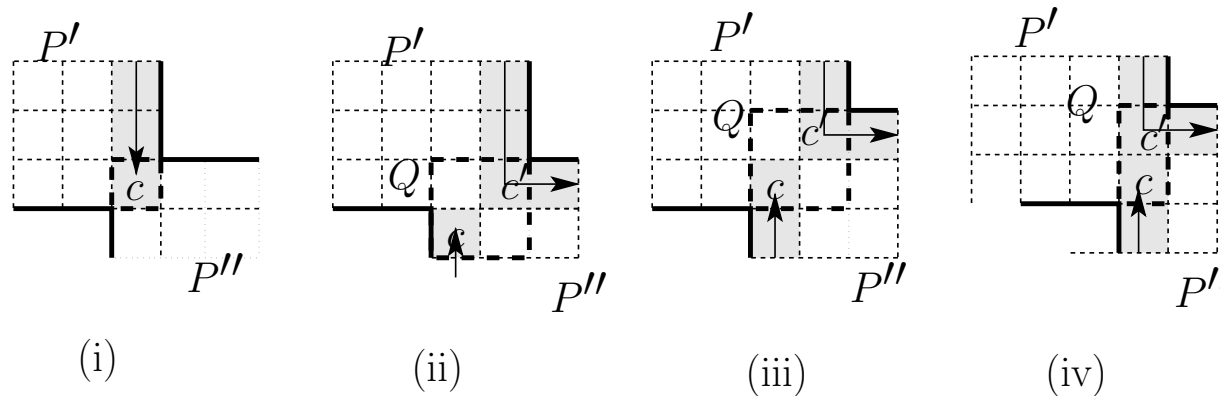


## Fall (ii),(iii): $S(P_i) \leq \frac{4}{3}C(P_i) - 2$

- At least  $4=|Q|$  steps less
- $S(P_i) = S(P') + S(P'') - 4$  and  $C(P_i) = C(P') + C(P'') - 4$
- Apply I.A. for  $P'$  and  $P''$

$$S(P_i) = S(P') + S(P'') - 4 \leq \frac{4}{3}C(P') + \frac{4}{3}C(P'') - 8$$

$$\leq \frac{4}{3}(C(P') + C(P'') - 4) - \frac{8}{3} < \frac{4}{3}C(P_i) - 2$$



# Summary SmartDFS

- Gridpolygons without holes ■
- Lower bound:  $\frac{7}{6}$  ■
- SmartDFS:  $\frac{4}{3}$  ■
- More sophisticated approach: approx.  $\frac{5}{4}$  ■
- Lower bound:  $\frac{20}{17}$  ■
- Optimale Offline Solution? ■