Online Motion Planning MA-INF 1314
Alternative cost measures!

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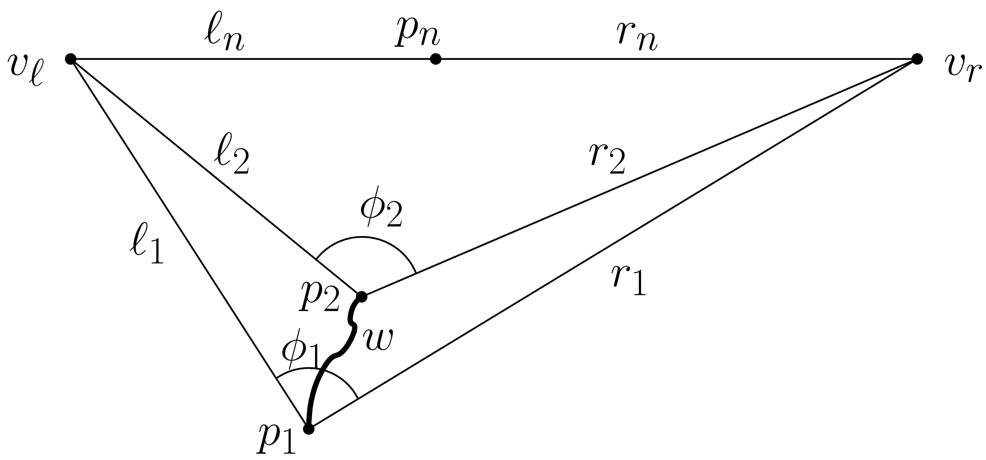
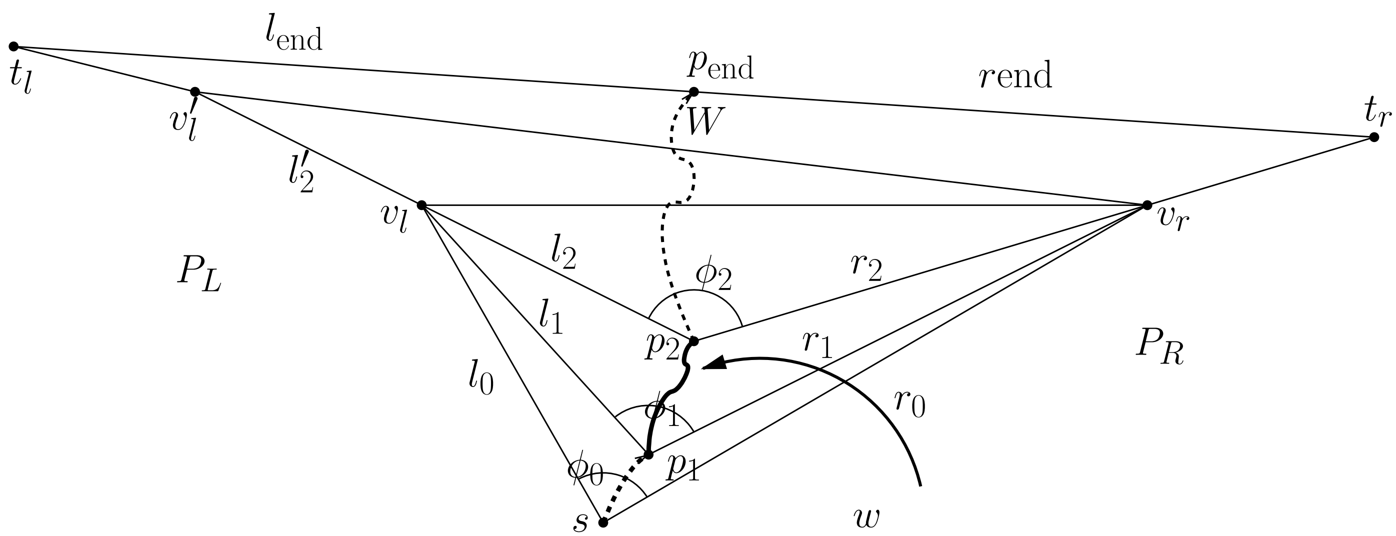
Rep.: Opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- Backward analysis!
- $\frac{|w|+K_{\phi_2}\cdot\ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w|+K_{\phi_2}\cdot r_2}{r_1} \leq K_{\phi_1}$
- Combine to one condition for w
- $|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \,,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$



Rep.: Opening angle$\pi \geq \varphi_0 \geq \pi/2$!

- Change of the reflex vertices! Sufficient!
- Change left side! Condition:
  $|w| \leq \min\{ K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \;,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \}$
- And also: $\frac{|w|+K_{\phi_2}\cdot(\ell_2+\ell'_2)}{(l_1+\ell'_2)} \leq K_{\phi_1}$

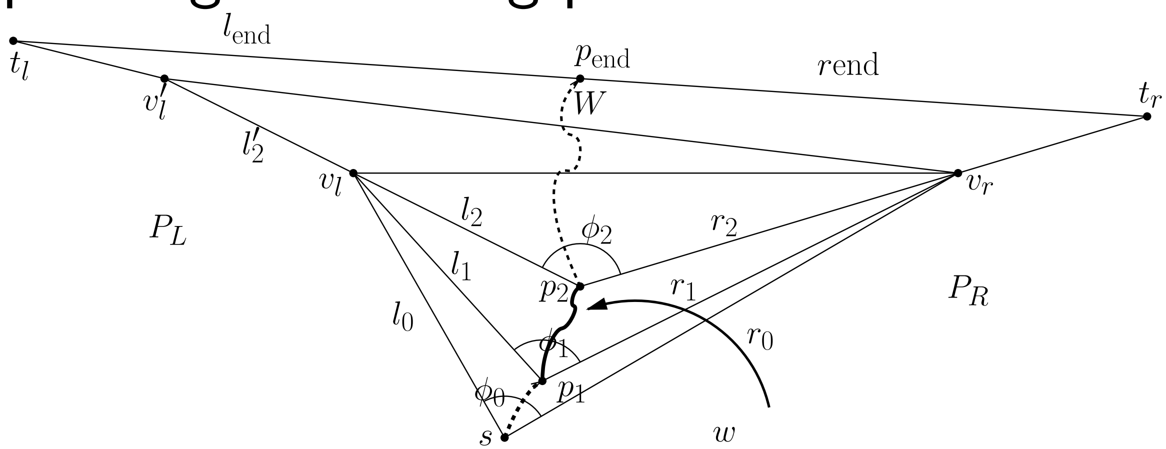


Rep.: Opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Lemma: Let S be a strategy, that searches for the target in a funnel with opening angle ϕ_2 for $\phi_2 \geq \frac{\pi}{2}$ with competitive ratio K_{ϕ_2}. This strategy can be extended to a strategy of ratio K_{ϕ_1} and opening angle ϕ_1 for $\phi_2 > \phi_1 \geq \frac{\pi}{2}$, if we guarantee

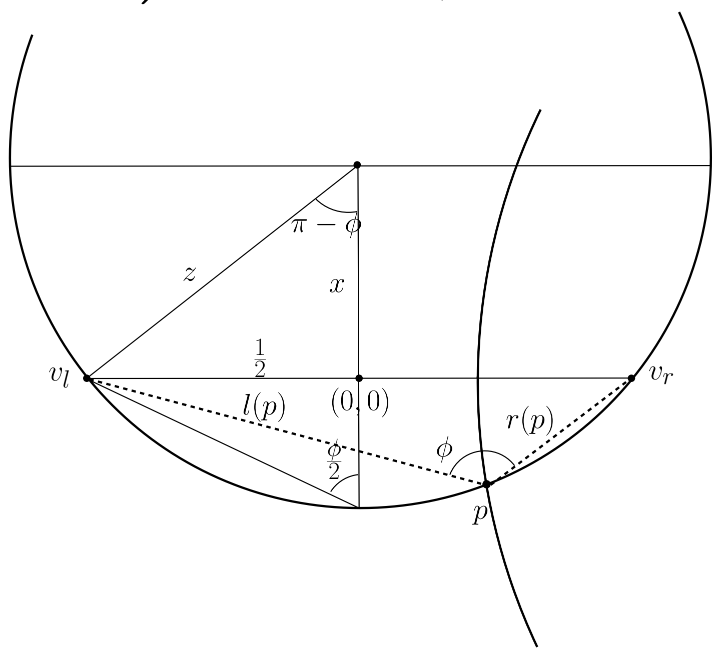
$$|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \;,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$$

for the corresponding connecting path.



Rep.: Opening angle W $\pi \geq \varphi_0 \geq \pi/2$!

- Equality: $K_{\phi_2}(\ell_2 - r_2) = K_{\phi_1}(\ell_1 - r_1)$, $A := K_{\phi_0}(\ell_0 - r_0)$
- Hyperbola: $\dfrac{X^2}{\left(\frac{A}{2K_\phi}\right)^2} - \dfrac{Y^2}{\left(\frac{1}{2}\right)^2 - \left(\frac{A}{2K_\phi}\right)^2} = 1$
- Circle: $X^2 + \left(Y + \dfrac{\cot\phi}{2}\right)^2 = \dfrac{1}{4\sin^2\phi}$

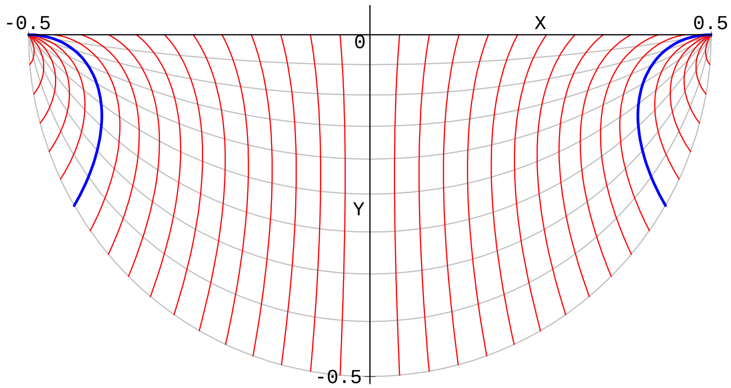


Rep.: Opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Intersection Hyp. Circle: Curve!!

- $$X(\phi) = \frac{A}{2} \cdot \frac{\cot \frac{\phi}{2}}{1+\sin\phi} \cdot \sqrt{\left(1+\tan\frac{\phi}{2}\right)^2 - A^2}$$
$$Y(\phi) = \frac{1}{2} \cdot \cot\frac{\phi}{2} \cdot \left(\frac{A^2}{1+\sin\phi} - 1\right)$$

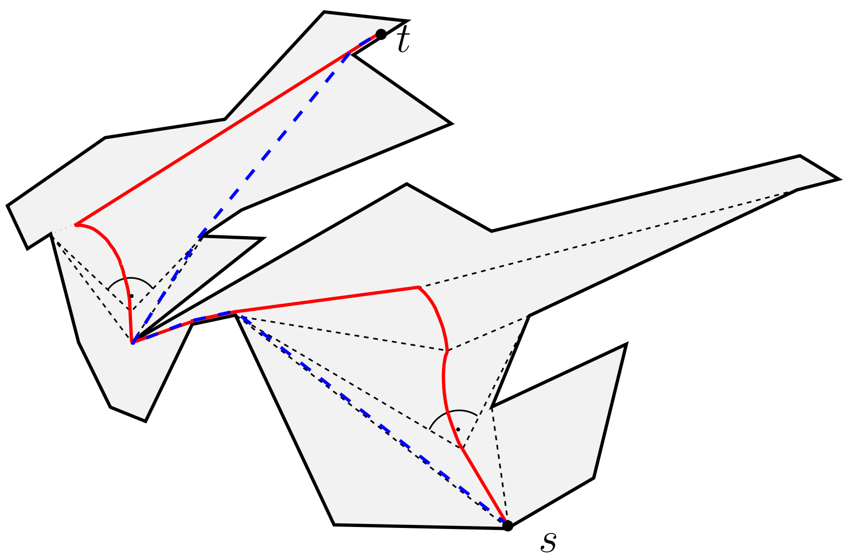
mit $A = K_{\phi_0}(\ell_0 - r_0)$



Opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Theorem: Searching for the target in a street polygon can be realized within a competitive ratio of $\sqrt{2}$.

- From $\varphi \geq \pi/2$ curve fulfils w condition, analysis/empriments!
- For smaller angles: $\sqrt{2}$ substitute for all K_ϕ



Optimal searchpath

- We have seen:
  Searching for a goal (polygon) in general not competitive
- Question: What is a good searchpath (for polygons)?
- Searching: Target point unknown!
- Offline-Searching: Environment is known
- Online-Searching: Environment unknown

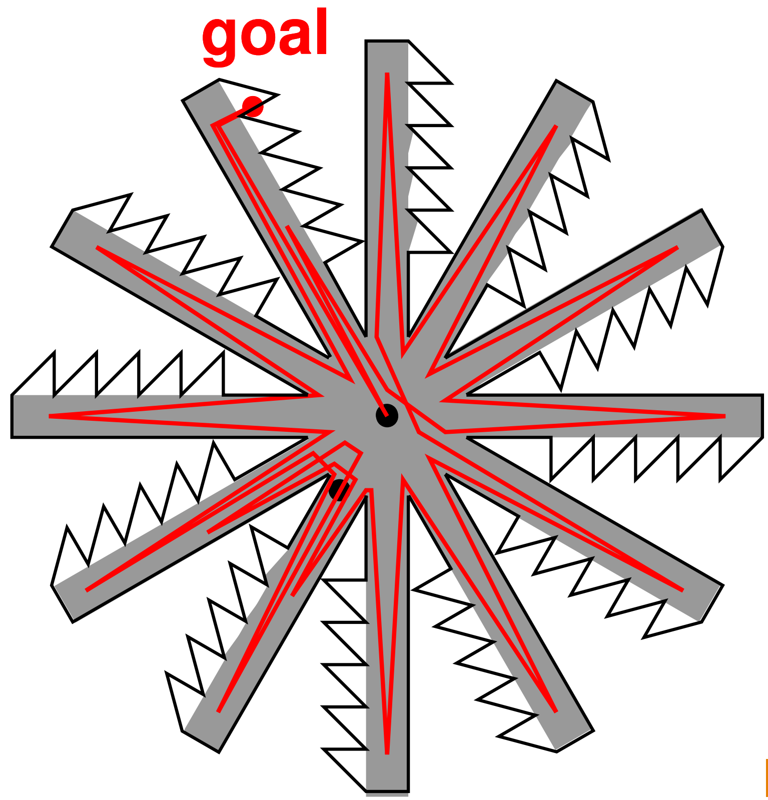
Quality measures!

- *Competitive ratio* of search strategy $\mathcal{A}$ in polygons:

$$C := \sup_{P} \sup_{p \in P} \frac{|\mathcal{A}(s,p)|}{|\mathsf{sp}(s,p)|}$$

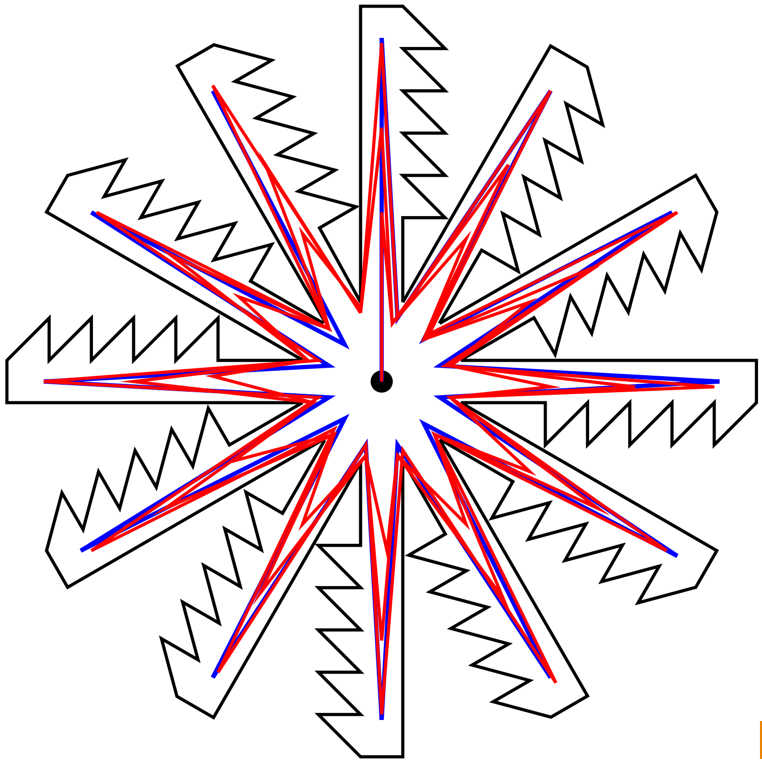
Optimal search path in polygons

- Competitive analysis:
- Agent with visibiltly
- Adversary forces any strategy to visit any corridor
- Optimal path is short
- $\Rightarrow$ Any strategy fails (not constant competitive)



Optimal search path in polygons

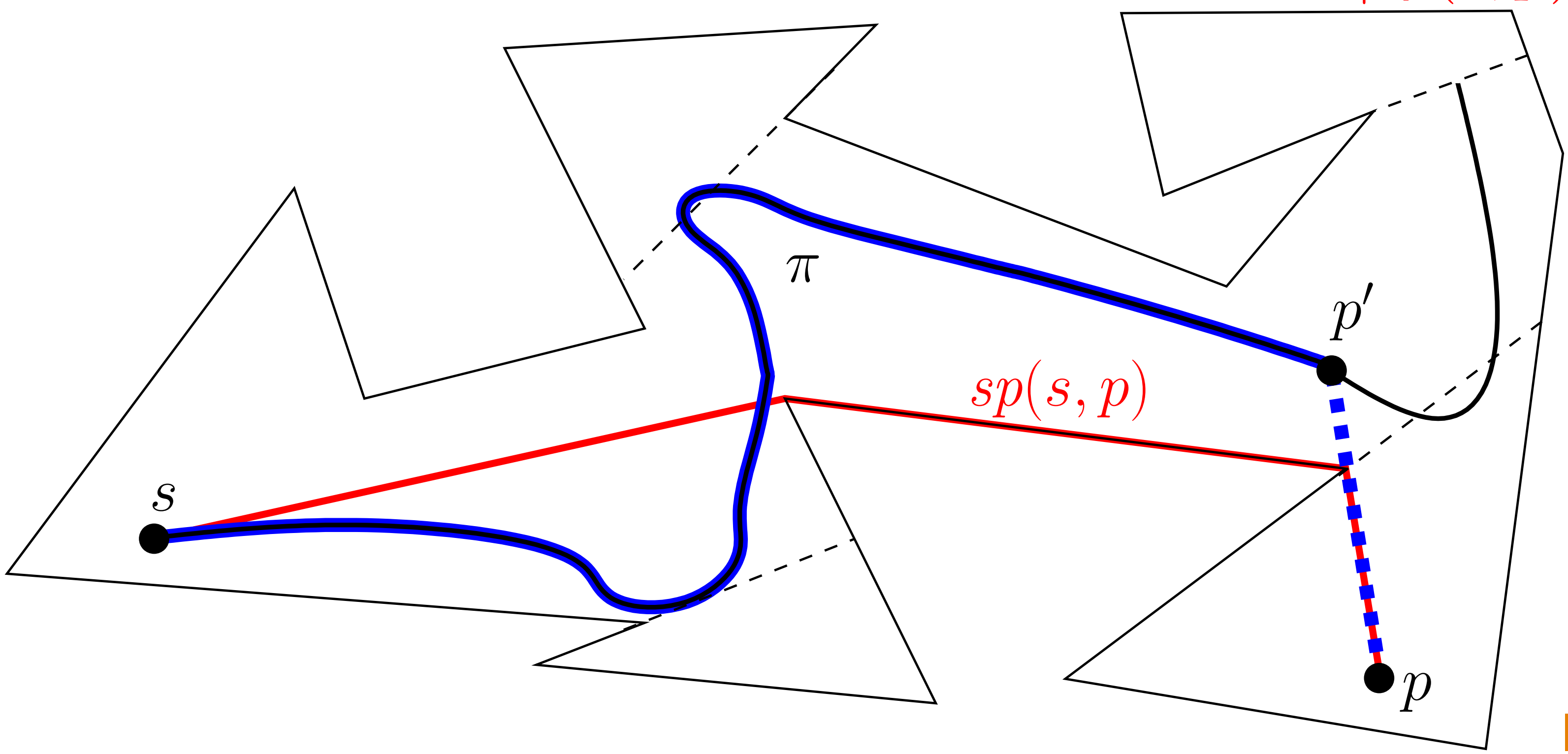
- Strat1:
  fully visit any corridor
- Strat2:
  visit all corr. depth $d = 1$
  visit all corr. depth $d = 2$
  visit all corr. depth $d = 4$ etc.



- Strat2 seems to be better:
  close targets s are visited earlier
- Can we give a measure?

Search ratio for polygons

π: **Searchpath**, quality for π: $\mathrm{SR}(\pi, P) = \max_{p \in P} \frac{|\pi_s^{p'}| + |p'p|}{|\mathrm{sp}(s,p)|}$



Search ratio in general

Given: Environment $\mathcal{E}$, Set of goals $\mathcal{G} \subseteq \mathcal{E}$

Graphs $G = (V, E)$: **Vertices** $\mathcal{G} = V$

Geometric Search $\mathcal{G} = V \cup E$

(Requirement: $\forall p \in \mathcal{E} : |\mathsf{sp}(s, p)| = |\mathsf{sp}(p, s)|$)

Search ratio of a search strategy $\mathcal{A}$ for $\mathcal{E}$:

$$\mathsf{SR}(\mathcal{A}, \mathcal{E}) := \sup_{p \in \mathcal{G}} \frac{|\mathcal{A}(s, p)|}{|\mathsf{sp}(s, p)|}$$

Optimal search ratio:

$$\mathsf{SR}_{\mathsf{OPT}}(\mathcal{E}) := \inf_{\mathcal{A}} \mathsf{SR}(\mathcal{A}, \mathcal{E})$$

Path with optimaler search ratio

- Graphs (offline): NP-hard
- Polygons (offline): ???
- Online: Approximation is possible

$\Rightarrow$ Goal: Approximate the path with opt. search ratio

Search ratio approximation

- *Competitive ratio* : $C := \sup_{\mathcal{E}} \sup_{p \in \mathcal{G}} \frac{|\mathcal{A}(s,p)|}{|\mathsf{sp}(s,p)|}$
- *Search ratio:* $\mathsf{SR}(\mathcal{A},\mathcal{E}) := \sup_{p \in \mathcal{G}} \frac{|\mathcal{A}(s,p)|}{|\mathsf{sp}(s,p)|}$
- *Optimal search ratio*: $\mathsf{SR}_{\mathsf{OPT}}(\mathcal{E}) := \inf_{\mathcal{A}} \mathsf{SR}(\mathcal{A},\mathcal{E})$
- Approximation: $\mathcal{A}$ *search-competitiv*

$$C_s := \sup_{\mathcal{E}} \frac{\mathsf{SR}(\mathcal{A},\mathcal{E})}{\mathsf{SR}_{\mathsf{OPT}}(\mathcal{E})}$$

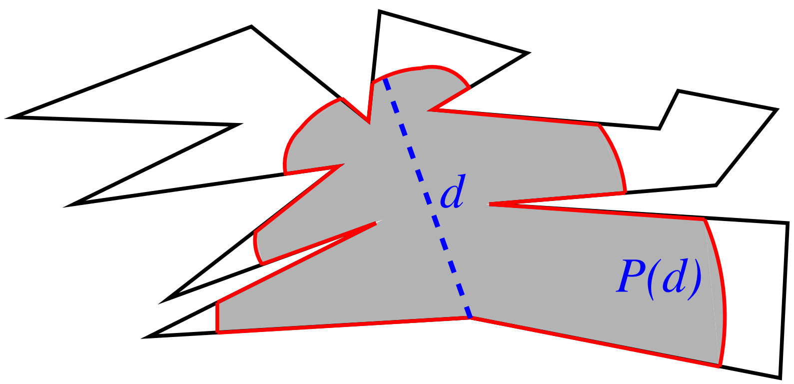
- Comparison not against SP, but against best possible SR

Depth-restricted exploration

Def. Exploration-Strategy Expl for $\mathcal{E}$ is called **depth-restrictable**, if we can derive a strategy $\mathrm{Expl}\,(d)$ such that:

- $\mathrm{Expl}\,(d)$ explores $\mathcal{E}$ up to depth $d \geq 1$
- return to s after the esploration
- $\mathrm{Expl}\,(d)$ is C-kompetitiv, i.e.,$\exists\, C \geq 1 :\ \forall \mathcal{E}$:

$$|\mathrm{Expl}\,(d)| \leq C \cdot |\mathrm{Expl}_{\mathrm{OPT}}(d)|\,.$$



Searchpath approximation

Algorithm

- Explore $\mathcal{E}$ by increasing depth: $\mathrm{Expl}\,(2^i)$ für $i = 1, 2, \ldots$

Lemma:

- Roboter without vision system
- Environment $\mathcal{E}$
- $\mathrm{Expl}_{\mathrm{ONL}}$: C-competitive, depth-restrictable, online exploration strategy for $\mathcal{E}$
  (d. h. $|\mathrm{Expl}\,(d)| \leq C \cdot |\mathrm{Expl}_{\mathrm{OPT}}(d)|$)

$\Rightarrow$ Algorithm gives $4C$-Approximation of optimal search path!

Searchpath approximation proof

$$|\Pi_{\text{Expl}_{\text{opt}}(d)}| \leq d \cdot (\mathsf{SR}(\Pi_{\text{opt}}) + 1) \quad (1)$$

$$\begin{aligned}
\mathsf{SR}(\Pi) \quad &\leq \quad \frac{\sum_{i=1}^{j+1} |\Pi_{\text{Expl}(2^i)}|}{2^j + \varepsilon} \\
&\underset{(\mathsf{Ugl.})}{\leq} \quad \frac{C}{2^j} \sum_{i=1}^{j+1} |\Pi_{\text{Expl}_{\text{opt}}(2^i)}| \underset{(1)}{\leq} \frac{C}{2^j} \sum_{i=1}^{j+1} 2^i \cdot (\mathsf{SR}(\Pi_{\text{opt}}) + 1) \\
&\leq \quad C \cdot \left(\frac{2^{j+2} - 1}{2^j} \right) \cdot (\mathsf{SR}(\Pi_{\text{opt}}) + 1) \leq 4C \cdot (\mathsf{SR}(\Pi_{\text{opt}}) + 1)
\end{aligned}$$

Applications

- **Corollay:**
  Trees: Exploration by DFS ($C = 1$) Online or Offline, depth restricted, simple

$\Rightarrow$ Searchpath approximation of factor 4

- Graphs: Online and Offline!
  CFS ($C = 4 + \frac{8}{\alpha}$) depth-restrictable!!
- But: Factor depends on rope length $(1 + \alpha)r$ by depth r
- CFS sometimes explores more than d (precisely $(1 + \alpha)d$)

$\Rightarrow$ $\mathrm{Expl}\,(d)$ not comparable to $\mathrm{Expl}_{\mathrm{OPT}}(d)$

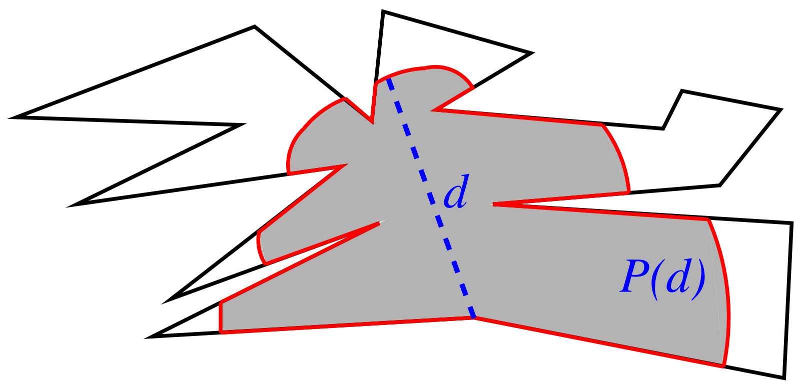
- Workaround: Compare $\mathrm{Expl}\,(d)$ with $\mathrm{Expl}_{\mathrm{OPT}}(\beta \cdot d)$

β-depth restricted exploration

Def. Exploration strategy Expl for $\mathcal{E}$ is denoted as **β-depth restrictable**, if we can derive a strategy $\mathrm{Expl}\,(d)$ such that:

- $\mathrm{Expl}\,(d)$ explores $\mathcal{E}$ only up to depth $d \geq 1$
- returns to the start s
- $\mathrm{Expl}\,(d)$ is C_β-competitiv, i.e., $\exists\, C_\beta \geq 1, \beta > 0 :\ \forall \mathcal{E}$:

$$|\mathrm{Expl}\,(d)| \leq C_\beta \cdot |\mathrm{Expl}_{\mathrm{OPT}}(\beta \cdot d)|\,.$$



Searchpath approximation

Theorem:

- Agent without vision
- Environment $\mathcal{E}$
- $\mathrm{Expl}:$ C_β-competitive, β-depth restrictable, online exploration strategy for $\mathcal{E}$, (i.e., $|\mathrm{Expl}\,(d)| \leq C_\beta \cdot |\mathrm{Expl}_{\mathrm{OPT}}(\beta \cdot d)|$)

$\Rightarrow$ Algorithm (exploration/double depth) gives a $4\beta C_\beta$-approximation of the optimal searchpath

Corollary: Unknown graphs, Algorithm with CFS is $4(1+\alpha)(4+\frac{8}{\alpha})$-approximation of optimal searchpath