Computer Science Lecture WS 2014/2015 Discrete and Computational Geometry

Randomized Algorithms for Geometric Structures

1. Define the conflict list for an interval, describe the randomized incremental version of Quick-Sort, and analyze the expected time complexity.

2. Prove that given a set N of n line segments with total k intersections and an i-element subset N^i of N, the expected number of trapezoids in the vertical trapezoidal decomposition $H(N^i)$ of N^i is $O(i + ki^2/n^2)$.

3. Define conflict relations between a newly inserted segment and the current trapezoidal decomposition $H(N^i)$, and describe how to insert a new segment.

4. Analyze the expected time of inserting a line segment into $H(N^i)$, and the total expected time for constructing the vertical trapezoidal decomposition.

5. Describe how to use a history graph to develop an on-line algorithm for the vertical trapezoidal decomposition and analyze the expected time complexity.

6. Please compare conflict graphs and history graphs.

7. Regarding the paper "Kenneth L. Clarkson, Kurt Mehlhorn, and Raimund Seidel Four, Results on Randomized Incremental Construction," define a configuration, conflict relations, and history, and give one example, e.g., vertical trapezoidal decomposition.

Chan's Randomized Technique and Geometric/Graph Dilations

1. What is a decision problem and what is an optimization problem? What is Chan's randomized technique? Explain the behind idea from the random-min algorithm, i.e., expected $O(Dr + E \log r)$ run time.

2. Give the precise definition of the notion of geometric/graph-theoretic dilation for a network!

3. How can we easily compute the dilation of a polygonal chain in polynomial time, e.g., $O(n^2)$ time? Summarize the algorithm. What kind of structural properties are helpful?

4. What is the geometric interpretation of a point on a polygonal chain and what is the relation between the geometric dilation of a polygon chain and the lower envelope of transformed cones? How can we the apply additively weight Voronoi diagram to develop a decision algorithm?

5. How to use Chan's randomized technique and the decision algorithm for the geometric dilation of a polygonal chain to develop an optimization algorithm?

Abstract Voronoi Diagrams

1. Define abstract Voronoi diagrams, describe the motivation, and list several examples. What is an admissible bisecting curve system?

2. Let $(S, \mathcal{J})$ be a bisecting curve system. Please prove that the following assertions are equivalent.

   - If p, q, and r are pairwise different sites in S, then $D(p,q) \cap D(q,r) \subseteq D(p,r)$ (Transitivity)
   - For each nonempty subset $S' \subseteq S$, $R^2 = \bigcup_{p \in s'} \overline{\mathrm{VR}(p, S')}$

3. Please argue that for checking an admissible bisecting curve system, it is enough to check all subset of 3 sites.

4. Define a conflict graph for the incremental construction of AVD, and prove that local test is enough, i.e., $e \cap \mathrm{VR}(t, R \cup \{t\}) = e \cap \mathrm{R}(t, \{p,q,r\})$, where $R \subseteq S$, $t \in S \setminus R$, and e is the Voronoi edge between $\mathrm{VR}(p,R)$ and $\mathrm{VR}(q,R)$.

5. Describe how to compute $V(R \cup \{s\})$ from $V(R)$, i.e., how to insert a new site S.

6. Describe how to update the conflict graph, i.e., computing $G(R \cup \{s\})$ from $G(R)$.

7. Describe how to transform the conflict-graph based algorithm into the history-graph based algorithm.

Geometric Duality, k-sets, and k^{th}-order Voronoi diagrams

1. How do we count the number of 2-partitions of an n-point set which can be separated by a straight line?

2. How do we enumerate all $O(n^2)$ 2-partitions of an n-point set which can be separated by a straight line?

3. What is a k-set? How do we bound the total number of $\leq k$=sets?

4. What are old and new Voronoi vertices of the k^{th}-order Voronoi diagram? What is the relation between the two kinds of Voronoi vertices? What are type-1 and type-2 Voronoi regions? How do the two kinds of Voronoi regions form from the previous-order Voronoi diagram?

5. Why can we derive a recursive formula for the complexity of the k^{th}-order Voronoi diagram? Please explain the reasons using old and new Voroinoi vertices, old and new Voronoi edges, and type-1 and type-2 Voronoi regions.

6. Please explain the iterative construction for the k^{th}-order Voronoi diagram.

Convexity and Lattice

1. What are affine subspace, affinely independent, affine combination, convex combination, convex hull? What is the relation between linear subspace and affine subspace?

2. What are Caratheodory's theorem, Radon's Lemma, and Helly's theorem? Please use Randon's lemma to prove Helly's theorem.

3. What are separation theorem for convex hulls and centerpoint theorem? Please prove the centerpoint theorem.

4. What is Minkowski's theorem? Please use an example to apply Minkowski theorem, e.g., forest visibility.

5. What is a general lattice and what is Minkowski's theorem for general lattices?

6. Please prove two-square theorem.