Discrete and Computational Geometry, SS 14
Exercise Sheet "4": Randomized Algorithms for Geometric Structures II
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 13th of May, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 10: Planar Convex Hull by Conflict Lists (4 Points)

Given a set N of n points in the plane, a convex hull $H(N)$ of N is a minimal convex polygon containing N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}.

1. Describe the insertion of S_{i+1}
2. Define a conflict relation between an edge of $H(N^i)$ and a point in $N \setminus N^i$
3. Prove the expected cost of inserting S^{i+1} to be $O(\frac{n}{i+1})$ and the expected cost of construction $H(N)$ to be $O(n \log n)$

Exercise 11: Triangulation (History Graph) (4 Points)

Given a set N of n points in the plane, a triangulation $H(N)$ of N is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to main the planarity. Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the history graph. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}.

1. Describe the parent and child relation in the history graph.
2. Describe the insertion of S_{i+1} using the history graph.
3. Prove the expected cost of inserting S^{i+1} to be $O(\frac{n}{i+1})$ and the expected cost of construction $T(N)$ to be $O(n \log n)$

Exercise 12: Planar Convex Hull by History Graph (4 Points)

Given a set N of n points in the plane, a convex hull $H(N)$ of N is a minimal convex polygon containing N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the history graph. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}.

1. Describe the parent and child relation in the history graph.
2. Describe the insertion of S_{i+1} using the history graph.
3. Prove the expected cost of inserting S^{i+1} to be $O(\frac{n}{i+1})$ and the expected cost of construction $T(N)$ to be $O(n \log n)$