Discrete and Computational Geometry Winter term 2016/2017
Exercise Sheet 03
University Bonn, Institute of Computer Science I

Deadline: Tuesday 08.11.2016, until 12:00 Uhr | Discussion: 14.11. - 18.11.

- *Please give your solutions directly to the tutor or put them in the postbox at LBH next to E.01 until the deadline. Write your names well visible and readable on the first page. If your solutions consists of multiple pages, make sure there are well connected.*
- *It is possible to submit in groups of up to three people.*

Aufgabe 1: Bisectors in additive-weighted Voronoi Diagrams (4 Punkte)

In the lecture we introduced additively-weighted Voronoi Diagrams. In such a diagram of a point set $S = \{p_1, p_2, \dots, p_n\}$, every point $p_i \in S$ is assigned an additive weight a_i. The Voronoi region is then given by $VR(p_i) = \{x \mid d(p_i, x) + a_i < d(p_j, x) + a_j \ \forall j \neq i, 1 \leq j \leq n\}$.

Prove that the bisector of two points with *different* weights in an additively weighted-Voronoi Diagram always forms a hyperbole.

Aufgabe 2: Voronoi Diagram of Line Segments (4 Punkte)

Consider a Voronoi-Diagram where the sites are not points, but line segments.

A bisector of two disjoint line segments consists of a combination of parabola and line segments. If following along the curve of a bisector the equation determining the curve changes $n-1$ times, we called it a bisector with n pieces.

If two line segments intersect at an endpoint, their bisector can also consist of an area touching the intersection.

a) What's the minimum and maximum number of pieces a bisector can have? Give an example of both. *Tip:* For a line segment ab, consider the area between the two lines intersecting ab orthogonally at a and b, respectively, as well as the intersections of the bisector curve with these strips.

b) Draw the Voronoi Diagram in picture 1 within the bordering rectangle. The parabola and line segments don't have to be incredibly precise, but the changes from bisector piece to piece should be clearly visible. Don't forget bisector areas.

c) Consider having a circular robot at starting point s that's supposed to reach destination t without colliding with any wall. Determine the maximum radius such a robot can have. Mark the critical bottleneck in the picture.

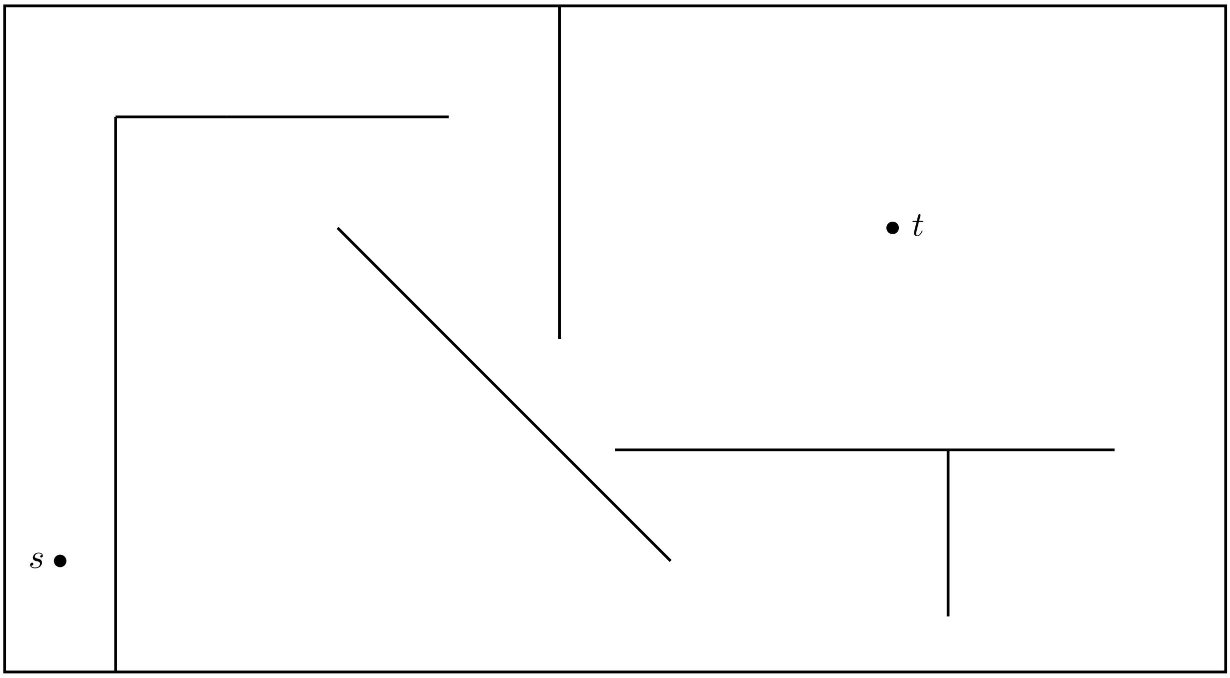

Abbildung 1: Ein Parcours.