Theoretical Aspects of Intruder Search

Course Wintersemester 2015/16
Escape Paths for the Intruder

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Reverse Situation

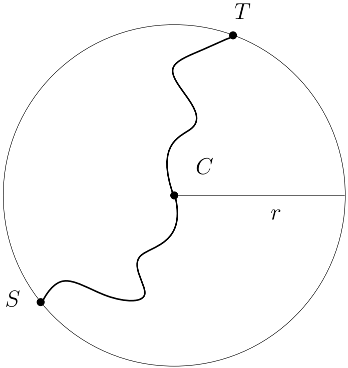
- Take over the position of the intruder
- Try to escape from an partially unknown environment
- The adversary manipulates the environment
- Leaev the area as soon as possible
- *Lost in a forest* Bellman 1956
- Escape paths for region R
- Single deterministic path
- Leave area from any starting point
- Adversary translates and rotates R
- Minimize the length of successful path
- Geometric argumentations
- Only known for few shapes

Simple examples

Obviously: The diameter of any region R is always an escape path!

Theorem 68: The shortest escape path for a circle of radius r is a line segment of length 2r.

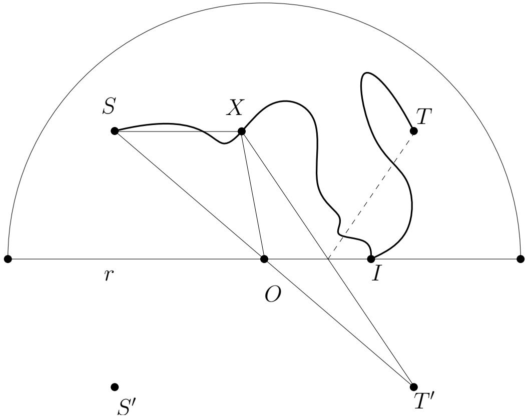
Proof: Assume there is a better escape path! Contradiction!



Also for semicircles

Theorem 69: The shortest escape path for a semicircle of radius r is a line segment of length 2r.

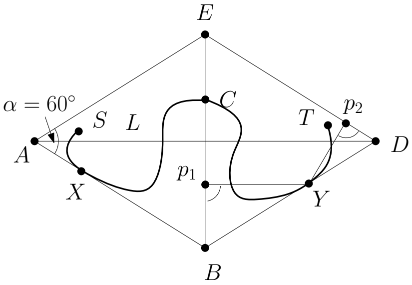
Proof: Assume there is a better escape path! Contradiction!



More generally for a rhombus with angle 60°

Theorem 70: The shortest escape path for rhombus of diameter L with angle $\alpha = 60^\circ$ is a line segment of length L.

Proof: Assume there is a better escape path! Contradiction!

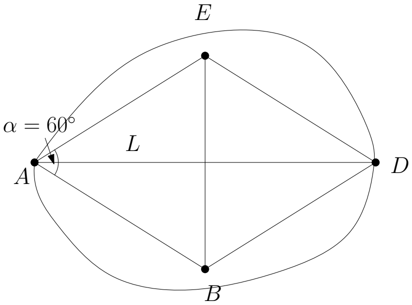


Fatness definition!

Definition: Fatness w.r.t. diameter! Rhombus-Fat!

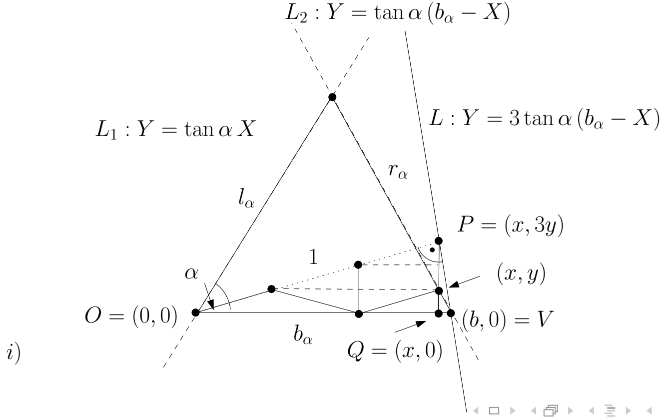
Corollary: The shortest escape path for rhombus-fat convex set of diameter L is a line segment of length L.

Proof: Assume there is a better escape path! Contradiction!

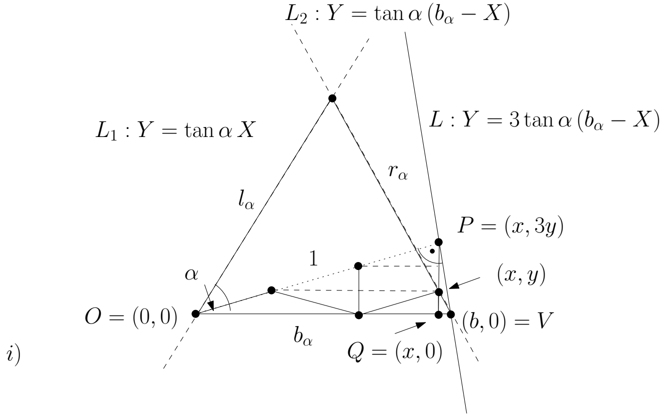


Convex = diameter?

- Equilateral triangle: Besicovitch
- Zig-Zag escape path with length ≈ 0.9812
- More generally from Coulton and Movshovich (2006)
- Isosceles triangle for α and b_α
- b_α is diameter!

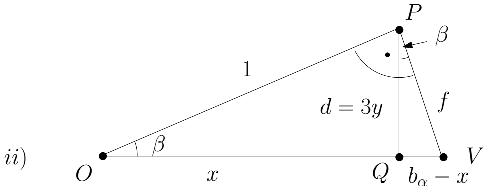


- Construct symmetric Zig-Zag path of small length
- Asssume length 1.



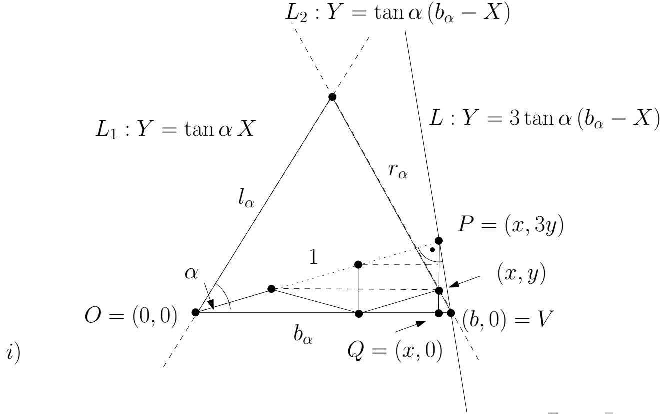
Convex = diameter?

- Extract triangle
- $\frac{1}{x} = \frac{b_\alpha}{1}$ $x = \frac{1}{b_\alpha}$



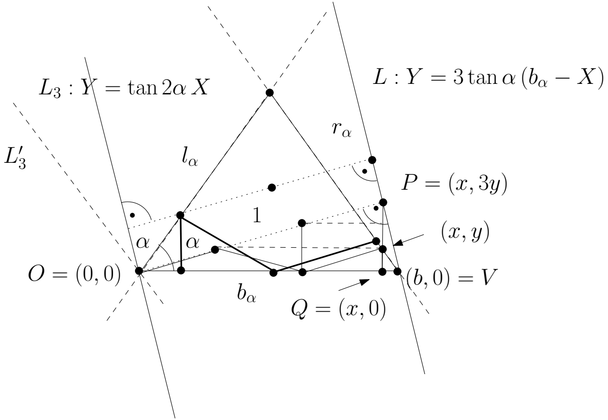
Finally we determine b_α:
$y = \tan\alpha\left(b_\alpha - \frac{1}{b_\alpha}\right)$ and $x = \frac{1}{b_\alpha}$ and $x^2 + (3y)^2 = 1$ which gives

$$b_\alpha = \sqrt{1 + \frac{1}{9\tan^2\alpha}}\,.$$



Further constraint for α

There should be no better Zig-Zag path for T_α!
Line $L_3 : Y = \tan(2\alpha)$ runs in parallel with L_2. This means $-3\tan\alpha = \tan 2\alpha$ or $\tan\alpha = \sqrt{\frac{5}{3}}$.



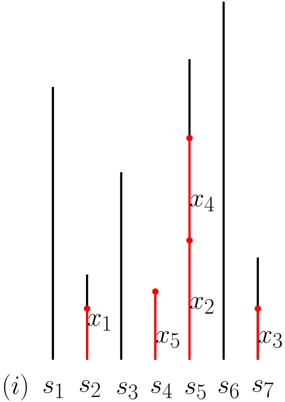
Besicovitch triangles

Theorem 71: For any $\alpha \in [\arctan(\sqrt{\frac{5}{3}}), 60^\circ]$ there is a symmetric Zig-Zag path of lenght 1 that is an escape path of T_α smaller than the diameter b_α.

- $b_\alpha = \sqrt{1 + \frac{1}{9\tan^2\alpha}}$
- $\alpha = 60^\circ$: $b_\alpha = \sqrt{\frac{28}{27}}$
- $b_\alpha := 1 \Longrightarrow \sqrt{\frac{27}{28}} < 1$ is Zig-Zag path length
- Optimality? Yes!

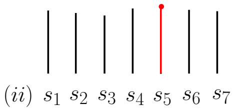
Different performance measures

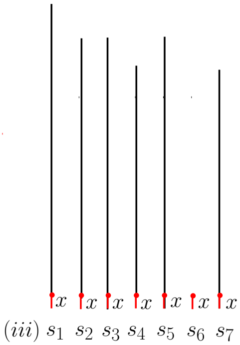
- Set L_m of m line segments s_i of unknown length $|s_i|$
- Dark corridors, escape, digging for oil
- Test corridors successively
- s_{j_1} up to a certain distance x_1, then s_{j_2} for another distance x_2 and so on



More information

- Assume distribution is known!
- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Extreme cases! Good strategies!





More information

- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Check i arbitrary segments with length f_i: $\min_i i \cdot f_i$ is the best strategy