

Discrete and Computational Geometry, WS1516  
Exercise Sheet “1”: Geometry Duality and Finding the  
Minimum  
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until **Wednesday 4th of November, 12:00 pm**.*
- *There is a letterbox in front of Room E.01 in the LBH building.*
- *You may work in groups of at most two participants.*

**Exercise 1:      Geometry Duality II      (4 Points)**

We define a geometry duality  $\Psi(\cdot)$  as follows ( $O$  denotes the origin)

- For a point  $p = (a, b) \in \mathbb{R}^2 \setminus O$ ,  $\Psi(p)$  maps to the line  $ax + by = 1$ .
- For a line  $L : ax + by = 1$ ,  $\Psi(L)$  maps to the point  $(a, b)$ .

Please prove the following.

For a point  $p \in \mathbb{R}^2 \setminus O$  and a line  $L$  that does not pass through  $O$ ,  $p$  and  $O$  are located in the same side of  $L$  if and only if  $\Psi(L)$  and  $O$  are located in the same side of  $\Psi(p)$ .

**Exercise 2:      Geometry Duality II      (4 Points)**

We define a geometry duality  $\Phi(\cdot)$  as follows

- For a point  $p = (a, b) \in \mathbb{R}^2$ ,  $\Phi(p)$  maps to the line  $y = ax - b$ .
- For a line  $L : y = ax - b$ ,  $\Phi(L)$  maps to the point  $(a, b)$ .

Please prove the following.

For a point  $p \in \mathbb{R}^2$  and a nonvertical line  $L$ ,  $p$  lies above  $L$  if and only if  $\Phi(L)$  lies above  $\Phi(p)$ .

**Exercise 3: Finding the Minimum (4 Points)**

Given  $r$  distinct numbers, let  $(a_1, a_2, \dots, a_r)$  be a random permutation of the  $r$  numbers. For  $i > 1$ , Let  $A_i$  be the event that  $a_i$  is smaller than all numbers in  $\{a_1, \dots, a_{i-1}\}$ . Please answer the following two questions.

- What is the probability  $\text{Prob}(A_i)$  of event  $A_i$ ?
- What is the value of  $\sum_{i=2}^r \text{Prob}(A_i)$ ?