Online Motion Planning, WT 13/14
Exercise sheet 9
University of Bonn, Inst. for Computer Science, Dpt. I

- *You can hand in your written solutions until Tuesday,* 07.01., 14:15*, in room E.06.*

Exercise 25: Search Ratio in bipartite graphs (4 points)

Suppose you are walking from vertex to vertex in the complete bipartite graph $K_{m,m}$ where each edge traversal costs 1. How does an optimal search path look like and what is its Search Ratio?

Reminder: The complete graph $K_{m,m} = (A \cup B, E)$ is defined as follows. The vertex set consists of two disjoint sets A, B of size m. An edge $e \in E$ exists exactly between every pair (a, b) of vertices, where $a \in A$ and $b \in B$.

Exercise 26: SearchRatio of a Grid (4 points)

When putting all the Christmas presents into Santa's bag, one christmas elf dropped it somewhere in their house, which is shown in Figure 1. As Christmas is drawing closer, Santa and the elves have to find it.

We assume that moving from one room to an adjacent one always takes one minute. Santa and the elves start in the room labelled s. Since there are so many elves, each of which follows its own route, we can assume that there is at least one elf that reaches the bag on a shortest path.

We are now interested in Santa's strategy. The question is how much longer it will take Santa to find the bag, compared to the first elf that will find it. In the following, x always refers to the length of a shortest path from the starting room to the room containing the bag.

Please turn the page!

Prove the following statements.

1. There is a search strategy for Santa that guarantees to find the bag after at most $7 \cdot x$ minutes, for any placement of the bag.

2. There is no strategy for Santa such that for every placement of the bag, Santa will find it after at most $5 \cdot x$ minutes.

The following exercises are optional:

3. There is no strategy for Santa such that for every placement of the bag, Santa will find it after at most $6 \cdot x$ minutes.

4. Provide a strategy π for Santa and prove that the SearchRatio of π is minimal.

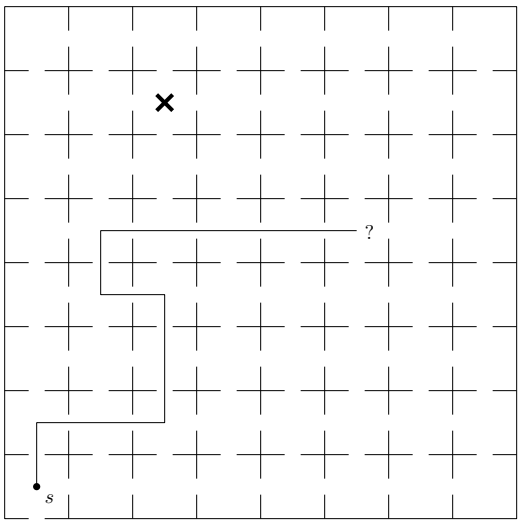

Figure 1: The house of the elves, having $8 \times 8 = 64$ rooms.

Exercise 27: CAB Algorithm Example (4 points)

Draw in the polygon in Figure 2 the path generated by the CAB algorithm.

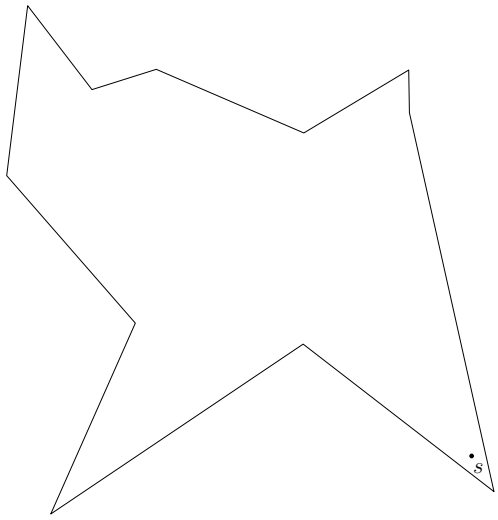

Figure 2: The polygon for the CAB algorithm.