

Discussion: 30.05. - 01.06.

Exercise Sheet 5

Exercise 5.1: Snake Lemma representant independence (4 Punkte)

Show that the choice of γ in the proof of the Snake-Lemma as given in the lecture is independent of the representatives α and β .

Exercise 5.2: Theorem of Mayer-Vietoris and Homologies (4 Punkte)

The Theorem of Mayer-Vietoris can be used to compute Homology of a topological space in a divide-and-conquer approach.

Recall: Let $K = K' \cup K''$ and $A = K' \cap K''$. By Mayer-Vietoris, there exists a long exact sequence

$$\dots \rightarrow H_p(K') \oplus H_p(K'') \rightarrow H_p(K) \rightarrow H_{p-1}(A) \rightarrow H_{p-1}(K') \oplus H_{p-1}(K'')$$

Proof that the following holds for the sequence above:

$$H_p(K) \cong \Im [H_p(K') \oplus H_p(K'')] \oplus \text{Ker} [H_{p-1}(A) \rightarrow H_{p-1}(K') \oplus H_{p-1}(K'')]$$

Exercise 5.3: Theorem of Mayer-Vietoris application (4 Punkte)

Apply Mayer-Vietoris to determine the Betti-numbers of the Double-torus by splitting it between both holes, which corresponds to cutting along the green line in the graphic below. As a first step, consider the what K , K' , K'' and A are in this szenario.

