

Discussion: 02.05. - 04.05.

## Exercise Sheet 3

### Exercise 3.1: Homeomorphism Examples

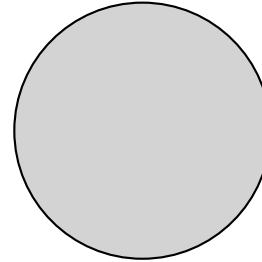
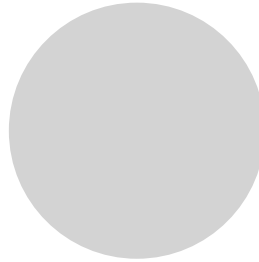
(4 Punkte)

Determine which of the following three objects are homeomorph. Prove your results.

circular disk (without boundary)



square disk (with boundary)



circular disk (with boundary)

### Exercise 3.2: Homeomorphism in Trees

(4 Punkte)

Consider two finite trees  $T_1$  and  $T_2$  without root or order. When are  $T_1$  and  $T_2$  isomorphic and when homeomorphic?

### Exercise 3.3: Homeomorphism by Bijection

(4 Punkte)

Prove the following statement from the lecture:

Let  $X$  be a compact space,  $Y$  be a Hausdorff space and  $f : X \rightarrow Y$  a continuous bijection. Then  $f$  is a homeomorphism.

*Hint:* Consider the inverse function  $g = f^{-1}$ . We need to show that  $g : Y \rightarrow X$  is continuous. It suffices to show that for any  $V \subseteq X$  it holds: if  $V$  is closed in  $X$  then  $g^{-1}(V) = f(V)$  is closed in  $T_2$ .