Discrete and Computational Geometry, SS 18
Exercise Sheet “7”: VC Dimension
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Thursday 21st of June**.
- *You may work in groups of at most two participants.*
- *You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.*

Exercise 19: Clarification: Proof of the lecture (4 Points)

On page 92 of the manuscript there are some arguments that state that the number of *different* cells for an arrangement of 6 wedges is not larger than 61! Clarify the statements by giving some additional information!

Exercise 20: Complexity of a wedge arrangement (4 Points)

A wedge is defined as two distinct rays in the plane, emanating from the same point p. We consider an arrangement $\mathcal{A}$ of n wedges. We are interested in the number of cells defined by $\mathcal{A}$. Suppose the set P of points in the plane where at least two wedges intersect has cardinality $|P| \leq k$ for some integer k.

We consider the following two cases.

a) Each point $p \in P$ is contained exactly two wedges.

b) Each point $p \in P$ can be contained in an arbitrary number of wedges.

Give matching upper and lower bounds for case a) (*Hint: use the Euler formula for planar graphs*) and try to give as good upper and lower bounds as possible for case b).

Exercise 21: VC Dimension in graphs (4 Points)

Suppose $G = (V, E)$ is an undirected, simple Graph[1]. We consider the set system $(X, \mathcal{F})$ where X equals the set E of edges of G, and where $\mathcal{F}$ is the set of *stars* in G. A *star* is the set of all edges adjacent to some vertex $v \in V$.

Prove an upper bound on the VC-Dimension of the set system $(X, \mathcal{F})$, and construct a graph G whose number of edges equals the upper bound, and where the edge set E of G is *shattered* by $\mathcal{F}$.

Which classical graph problem cooresponds to constructing a minimal transversal of the dual set system $(\tilde{X}, \tilde{\mathcal{F}})$, $\tilde{X} := \mathcal{F}$, $\tilde{\mathcal{F}} := \{\{F \in \tilde{X} | x \in F\} | x \in X\}$?

[1] A Graph $G = (V, E)$ is called simple if G contains no edge (v, v), $v \in V$, and at most one edge (v, u) for each pair u, v of vertices