Discrete and Computational Geometry, SS 18
Exercise Sheet "10": Trapezoidal decompositions
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Thursday 12th of July**.
- *You may work in groups of at most two participants.*
- *You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.*

Exercise 28: Building a trapezoidal map (4 Points)

Apply the procedure to create a trapezoidal decomposition described in the lecture. Given the following point set S in $\mathbb{R}^2$.

$$S = \{(-4,-4),(0,-3),(0,3),(3,-6),(6,1)\}$$

a) Create (per Hand) the euclidean Voronoi Diagram $V(S)$ of S.

b) Create (per Hand) the trapezoidal decomposition $T(V(S))$, such that you can locate a query point p from $[-10,10]\times[-10,10]$ in the voronoi regions.

c) Create a DAG $D(V(S))$ for $T(V(S))$ as a data structure for point localisation using the incremental procedure from the lecture. Add the line segments of the Voronoi Diagram from the left to the right (ordered by the x-coordinate of their left endpoint, breaking ties by x-coordinate of the right endpoint).

d) Mark the query path for the following 3 queries in $D(V(S))$:
$p_1 = (1,1)$, $p_2 = (9,-8)$, $p_3 = (-1,0)$.
Which voronoi regions do you get as a result?

Exercise 29: Properties of the map (4 Points)

Prove the following propositions:

a) Every face f of a trapezoidal decomposition $T(S)$ of a set S of n line segments in general position[1] is bordered by one or two vertical edges and exactly two non-vertical edges.
Tip: First prove that every f is convex.

b) The trapezoidal decomposition $T(S)$ of a set S of n line segments in general position[1] consists of at most $6n+4$ vertices and at most $3n+1$ trapezoids.

Exercise 30: Triangulation of simple polygons (4 Points)

We have a look at the randomized incremental construction for computing the trapezoidal map of a simple polygon.

a) Briefly repeat the main idea of exploiting the fact that the edges of a polygon are connected.

b) For the final triangulation of the polygon it remains to show that very special polygons can be triangulated in linear time. The polygon is subdivided into two monotone chains, where one chain is a single edge. Describe a linear time algorithm.

[1] The line segments are in general position, if they intersect only at endpoints and no two different endpoints (from the same or different line segments) have the same x-coordinate.