Online Motion Planning MA-INF 1314
Searching

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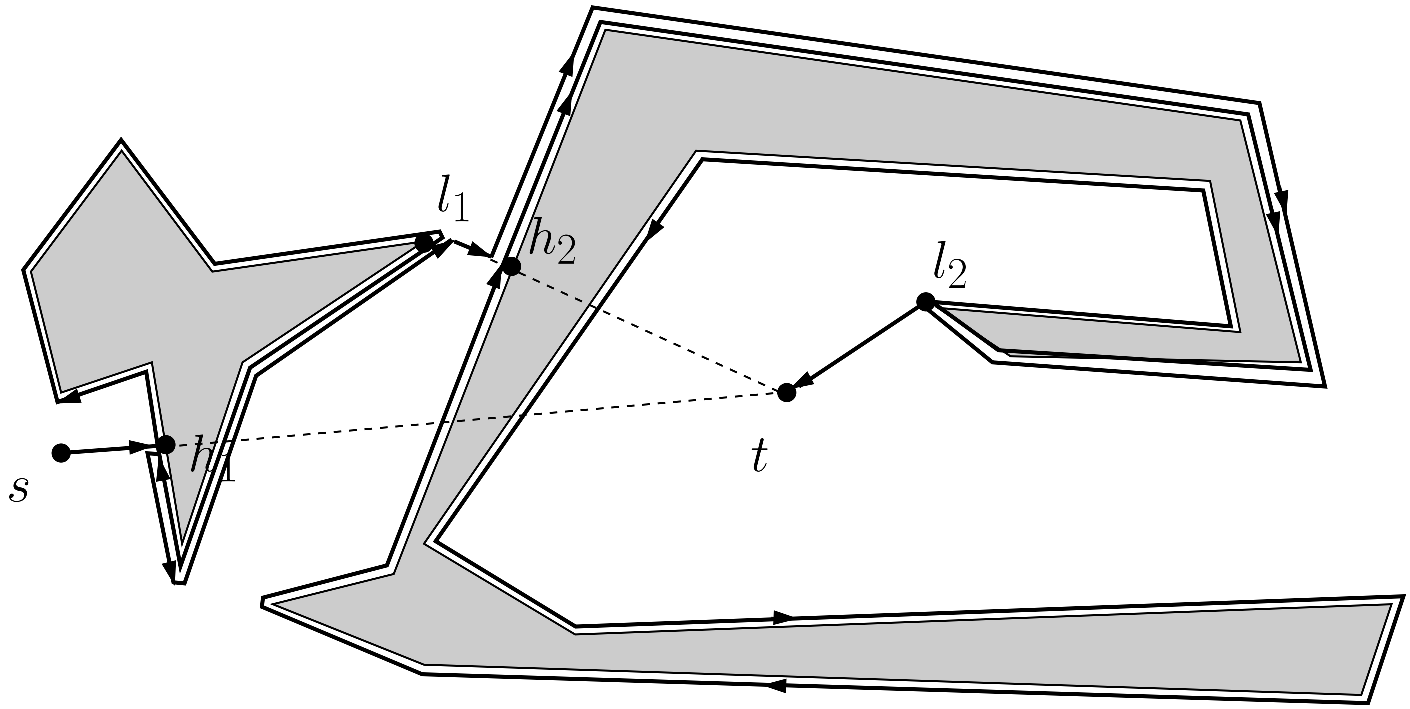
Rep.: Navigation

- Touch sensor, Target coordinates, Start s, Target t, Storage, Sojourner
- Actions:
  - Move toward the target
  - Move along the boundary
  - Sequence of Leave-Points l_i, Hit-Points h_i

Rep.: BUG1 Strategy: Lumelsky/Stepanov

Toward target, surround obstacle, best leave point, toward target!

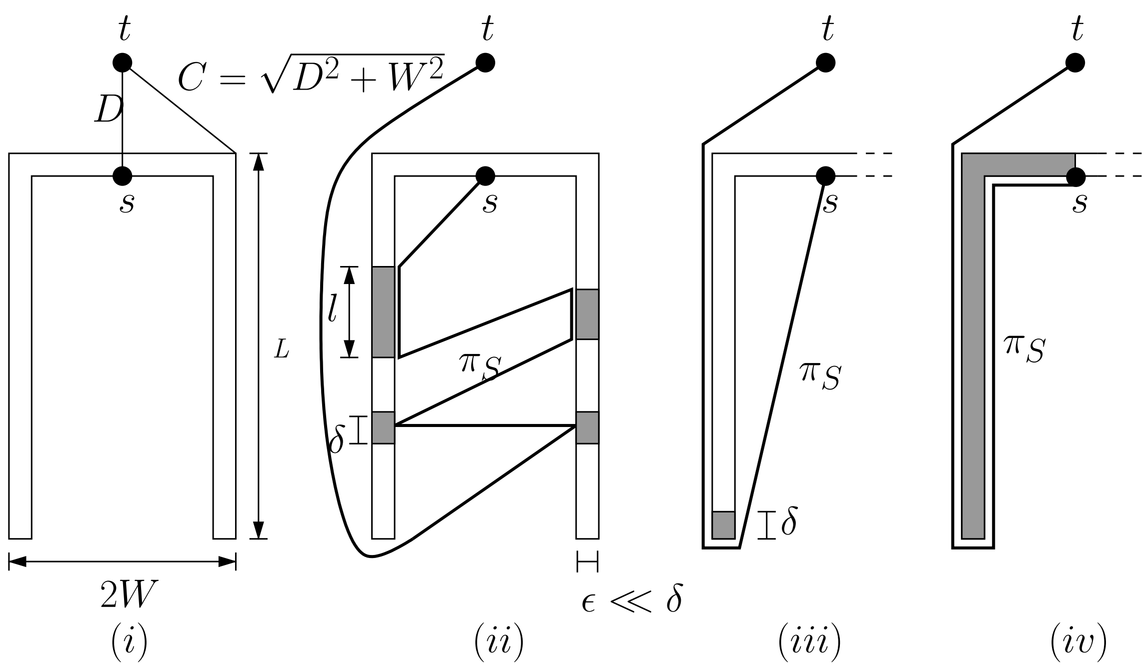


Rep: Analysis BUG1 Strategy

- **Theorem** Strategy Bug1 is correct!
- **Theorem** Successful Bug1-path $\Pi_{\texttt{Bug1}}$ from start s to target t: $|\Pi_{\texttt{Bug1}}| \leq D + \frac{3}{2}\sum_i \texttt{UP}_\texttt{i}$.
- **Theorem** For any strategy S, for arbitrary large $K > 0$, there exists examples for any $D > 0$, such that for any arbitrarily small $\delta > 0$ we have: $|\Pi_S| \geq K \geq D + \sum \texttt{UP}_\texttt{i} - \delta$.
- **Korollar** Bug1 is $\frac{3}{2}$-competitive against any *online* strategy

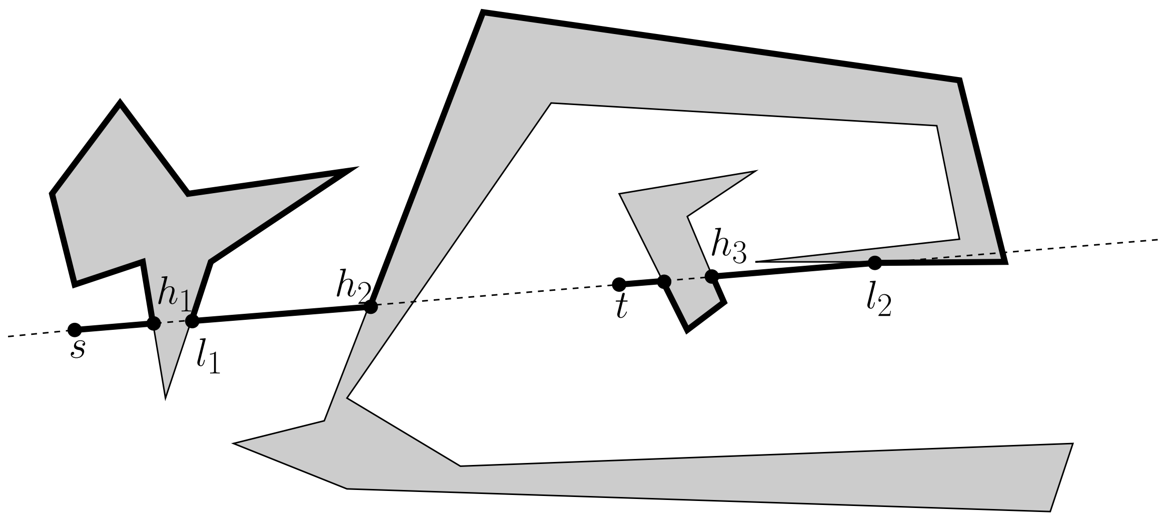
Rep. LB: $|\Pi_S| \geq K \geq D + \sum \texttt{UP}_\texttt{i} - \delta$

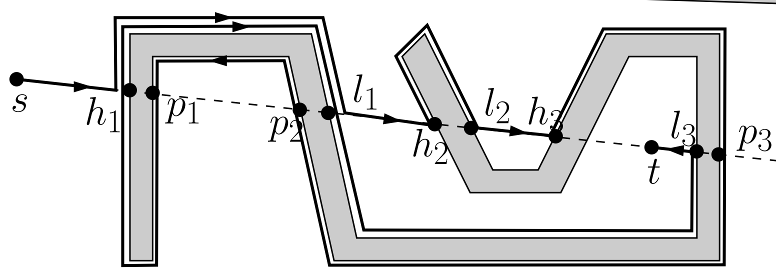
- Virtual horse-shoe , width $2W$, thickness $\epsilon \ll \delta$, length L, dist. D
- Virtual gets concrete by touch
- Roughly surround any obstacle, by any strategy!



Rep.: BUG2 Strategy

Line G passing st, toward target, surround obstacle, shorter distance on G, toward target!



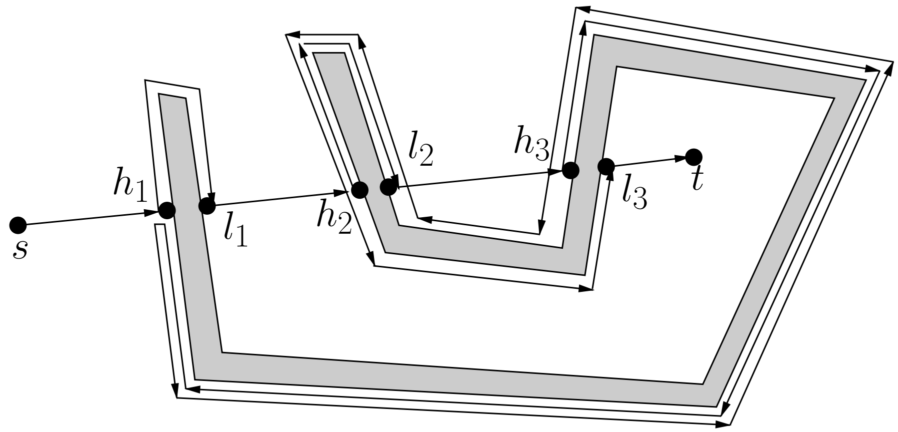


Rep.: Analysis BUG2 Strategy

- **Lemma** Let n_i denote the number of intersection of G with relevant obstacle P_i. Bug2 meets any point on P_i at most $\frac{n_i}{2}$ times.
- **Corollar** Bug2 is correct!
- **Theorem** Bug2-path Π_{Bug2} from s to t. We have: $|\Pi_{\mathrm{Bug2}}| \leq D + \sum_i \frac{n_i \mathtt{UP_i}}{2}$.

Rep.: Change I

Change I, use former Leave/Hit Points once for !

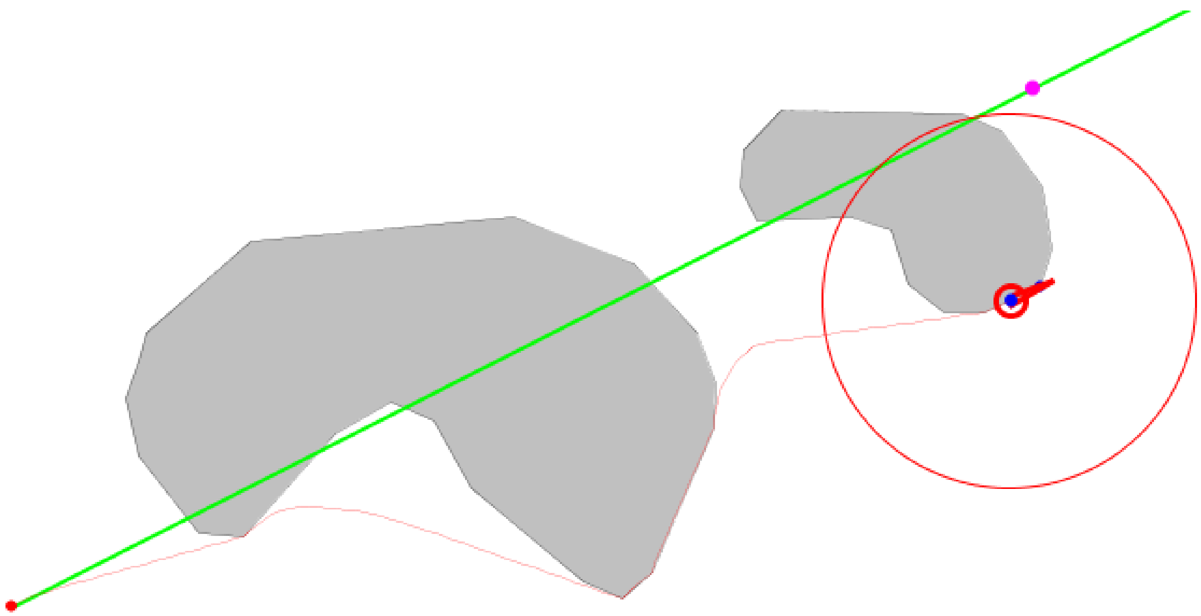


Theorem: Change I requires at most path length $|\Pi_{\text{ChangeI}}| \leq D + 2\sum_i \texttt{UP}_\texttt{i}$. This is a tight bound!

Exercise!

Different models

- Sensor with range: Circle aroung curr. point
- Short-cut for BUG2: VisBug
- Many others

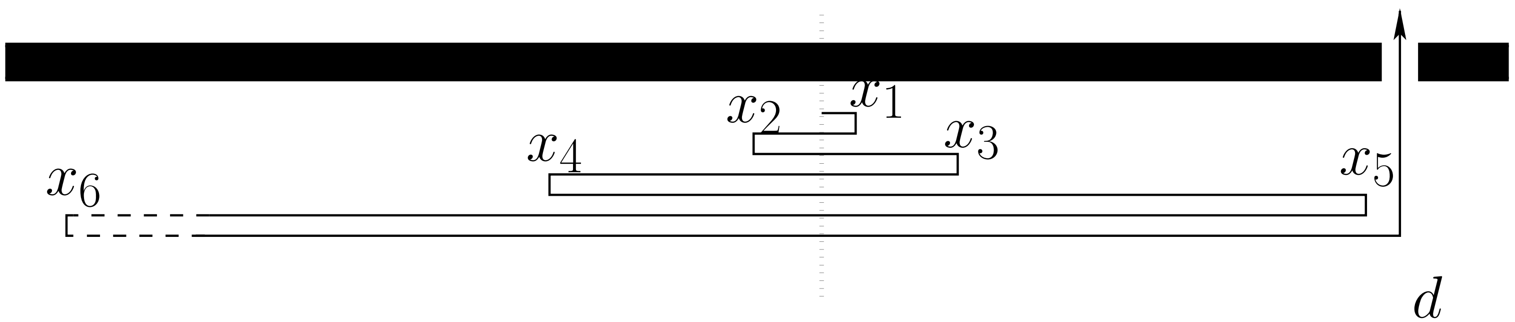


Searching for a goal!

- Coordinates of the target unknown: Searching vs. Navigation
- Polygonal environment
- Full sight: Visibility polygon
- **Def.** Let P be a simple polygon and r a point with $s \in P$. The visibility polygon of r w.r.t. P, $\mathrm{Vis}_P(r)$, is the set of all points $q \in P$, such that the segment $\overline{rq}$ is fully inside P.
- Alg. Geom.: Compute in $O(n)$ time! Offline!

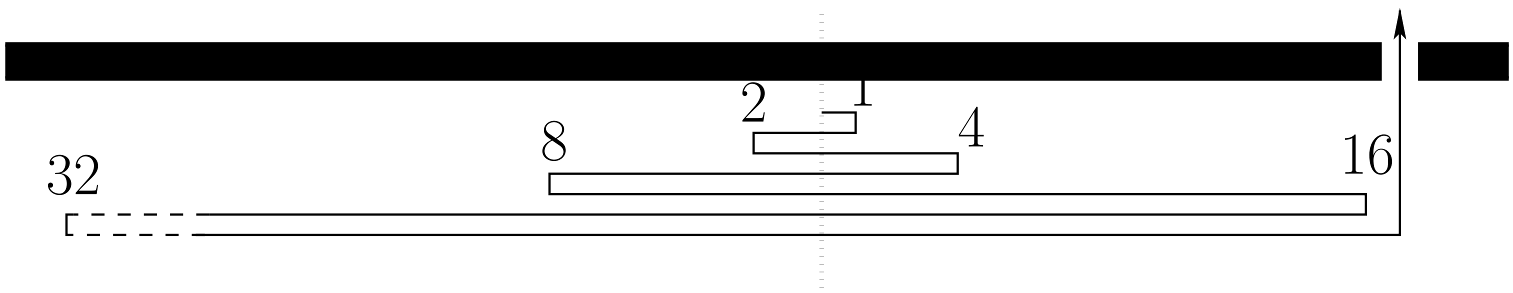
Corridors (without sight)

- 2-ray search: Find door along a wall!
- Compare to shortest path to the door, competitive?
- Reasonable strategy: Depth x_1 right, depth x_2 left and so on
- Start-situation: $2x_1 \geq C\epsilon$, for any $C > 0$ ex. ϵ
- Additive constant or goal is at least step 1 away!
- Local worst-case, not visited at d, once back!
- Find strategy, such that: $\sum_{i=1}^{k+1} 2x_i + x_k \leq C x_k$



Corridors

- Worst-case, not visited at d, once back!
- Find strategy, such that: $\sum_{i=1}^{k+1} 2x_i + x_k \leq C x_k$
- Minimize: $\frac{\sum_{i=1}^{k+1} 2x_i + x_k}{x_k} = 1 + 2\frac{\sum_{i=1}^{k+1} x_i}{x_k}$
- $x_i = 2^{i-1}$, gives ratio $C = 9$
- Proof: Blackboard!



Theorem Opt. of exponential solution: Gal 1980

- Strategy: Sequence $X = f_1, f_2, \ldots$
- Minimize functional $F_k(f_1, f_2, \ldots) := \frac{\sum_{i=1}^{k+1} f_i}{f_k}$ for all k
- More precisely $\inf_Y \sup_k F_k(Y) = C$ und $\sup_k F_k(X) = C$
- In general: Functional F_k continuous/unimodal: Unimodal: $F_k(A \cdot X) = F_k(X)$ and $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$
- Some other helpful conditions!
- I.e.: $F_{k+1}(f_1, \ldots, f_{k+1}) \geq F_k(f_2, \ldots, f_{k+1})$
- **Theorem** Exponential function minimizes F_k:

$$\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$$

mit $A_a = a^0, a^1, a^2, \ldots$ und $a > 0$.

Example: Exponential function

- $F_k(f_1, f_2, \ldots) := \frac{\sum_{i=1}^{k+1} f_i}{f_k}$ for all k.
- Unimodal $F_k(A \cdot X) = F_k(X)$ and $F_k(X+Y) \leq \max\{F_k(X), F_k(Y)\}$?
- $\frac{\sum_{i=1}^{k+1} A \cdot f_i}{A \cdot f_k} = \frac{\sum_{i=1}^{k+1} f_i}{f_k}$
- $F_k(X+Y) \leq \max\{F_k(X), F_k(Y)\}$?
- Follows from $\frac{a}{b} \geq \frac{c}{d} \Leftrightarrow \frac{a+c}{d+b} \leq \frac{a}{b}$
- Simple equivalence !
- Optimize: $f_k(a) := \frac{\sum_{i=1}^{k+1} a^i}{a^k}$
- Minimized by $a = 2$

Theorem Gal 1980

If finctionaly F_k has the following properties:

i) F_k is continuous,

ii) F_k is unimodal: $F_k(A \cdot X) = F_k(X)$ and $F_k(X+Y) \leq \max\{F_k(X), F_k(Y)\}$,

iii) $\liminf_{a \mapsto \infty} F_k \left(\frac{1}{a^{k+i}}, \frac{1}{a^{k+i-1}}, \dots, \frac{1}{a}, 1\right) = \liminf_{\epsilon_{k+i}, \epsilon_{k+i-1}, \dots,, \epsilon_1 \mapsto 0} F_k \left(\epsilon_{k+i}, \epsilon_{k+i-1}, \dots, \epsilon_1, 1\right)$,

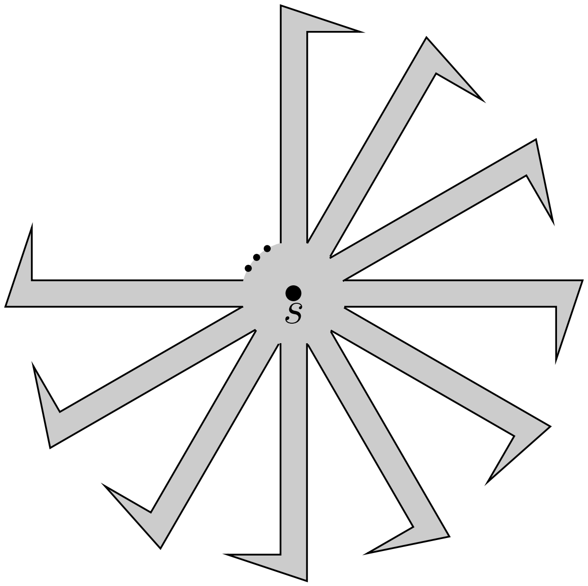
iv) $\liminf_{a \mapsto 0} F_k \left(1, a, a^2, \dots, a^{k+i}\right) = \liminf_{\epsilon_{k+i}, \epsilon_{k+i-1}, \dots \epsilon_1 \mapsto 0} F_k \left(1, \epsilon_1, \epsilon_2, \dots, \epsilon_{k+i}\right)$,

v) $F_{k+1}(f_1, \dots, f_{k+i+1}) \geq F_k(f_2, \dots, f_{k+i+1})$.

Then:$\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$ with $A_a = a^0, a^1, a^2, \dots$ and $a > 0$.

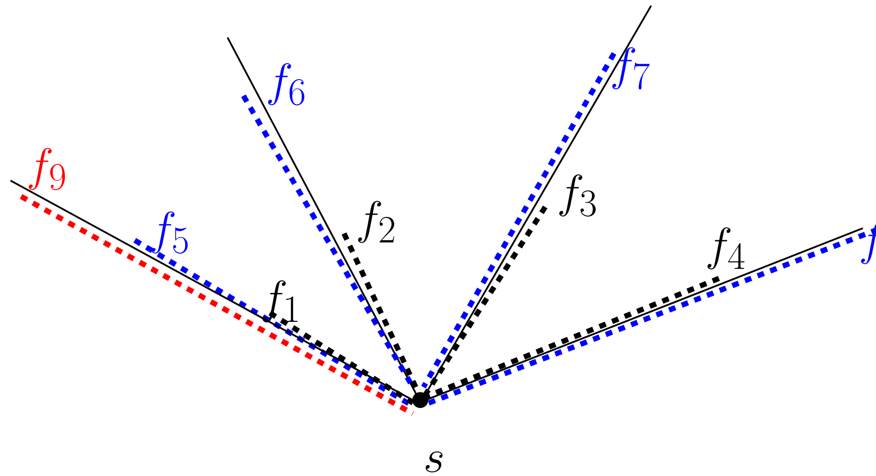
Application m-ray search

- Arbitrary m, not competitive, Fig.!
- $2m-1$ vs. 1!
- Fixed m, infinite rays!
- Ass.: Rays in fixed order and increasing depth
- Tupel (f_j, J_j): depth, next visit!



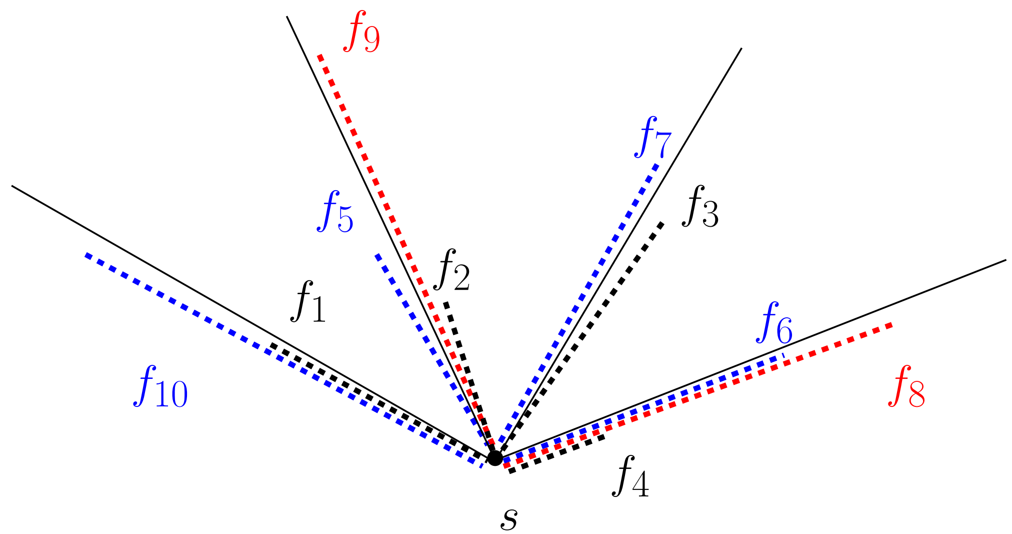
Applicatiot m-ray search

- Ass.: (f_j, J_j), $J_j = j + m$, $f_j \geq f_{j-1}$
- Visit rays in fixed order, increasing depth
- $F_k(f_1, f_2, \ldots) := \frac{f_k + 2\sum_{i=1}^{k+m-1} f_i}{f_k}$ for all k
- (Gal) Exp.-function minimizes F_k: $\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$ with $A_a = a^0, a^1, a^2, \ldots$ and $a > 1$, optimal $a = \frac{m}{m-1}$
- Ratio: $C = 1 + 2m\left(\frac{m}{m-1}\right)^{m-1}$ opt.



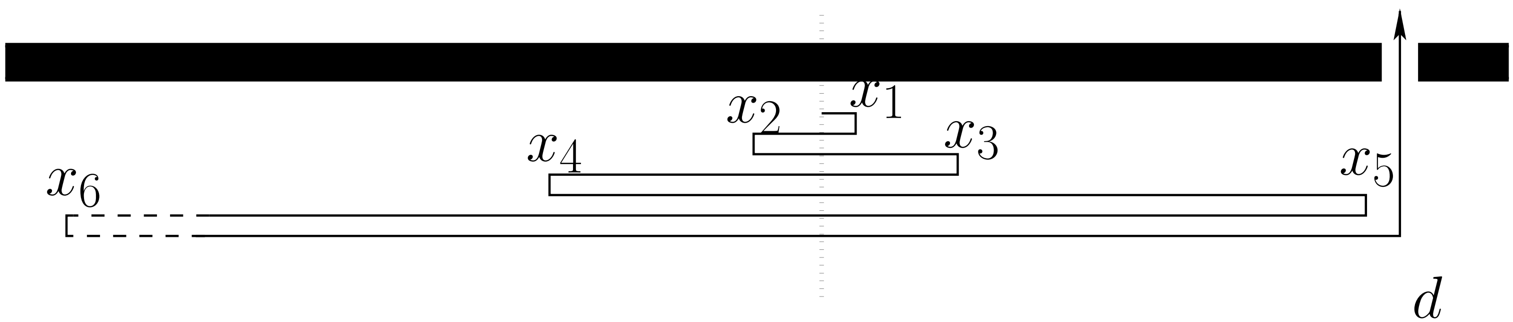
m-ray search

- **Lemma** There is an optimal m-ray search strategy $(f_1, f_2, \ldots)$ that visits the rays in a fixed order and with increasing depth.
- periodic and monotone: (f_j, J_j), $J_j = j + m$, $f_j \geq f_{j-1}$
- Second part: Proof blackboard! Change strategy! Conditions!



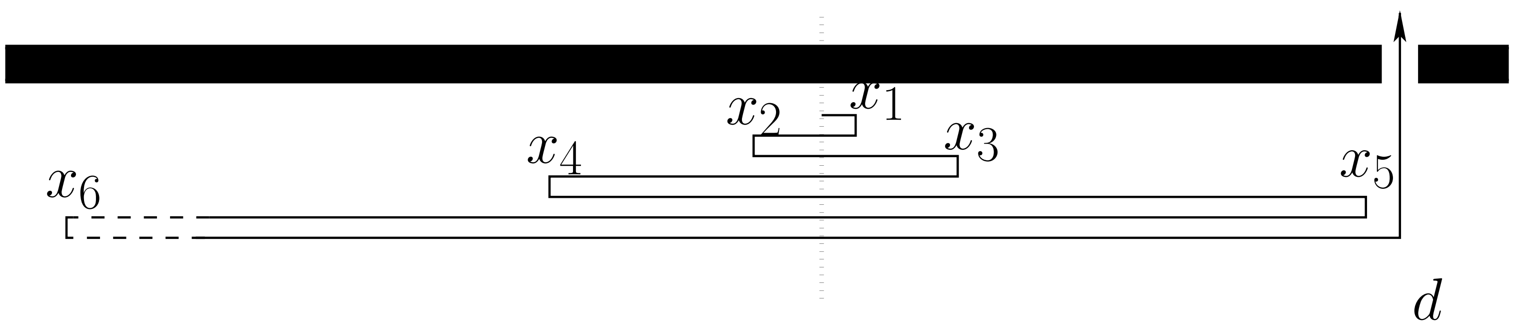
Other approach: Optimality for equations!

- Reasonable strategy, ratio: $\frac{\sum_{i=1}^{k+1} 2x_i + x_k}{x_k} = 1 + 2\frac{\sum_{i=1}^{k+1} x_i}{x_k}$
- Ass.: C optimal, $\frac{\sum_{i=1}^{k+1} x_i}{x_k} \leq \frac{(C-1)}{2}$
- There is strategy $(x'_1, x'_2, x'_3 \ldots)$ s. th. $\frac{\sum_{i=1}^{k+1} x'_i}{x'_k} = \frac{(C-1)}{2}$ for all k
- Monotonically increasing in x'_j $(j \neq k)$, decreasing in x'_k
- First k with: $\frac{\sum_{i=1}^{k+1} x_i}{x_k} < \frac{(C-1)}{2}$, decrease x_k
- $\frac{\sum_{i=1}^{k} x_i}{x_{k-1}} < \frac{(C-1)}{2}$!, x_{k-1} decrease etc., monotonically decreasing sequence, bounde, converges! Non-constructive!



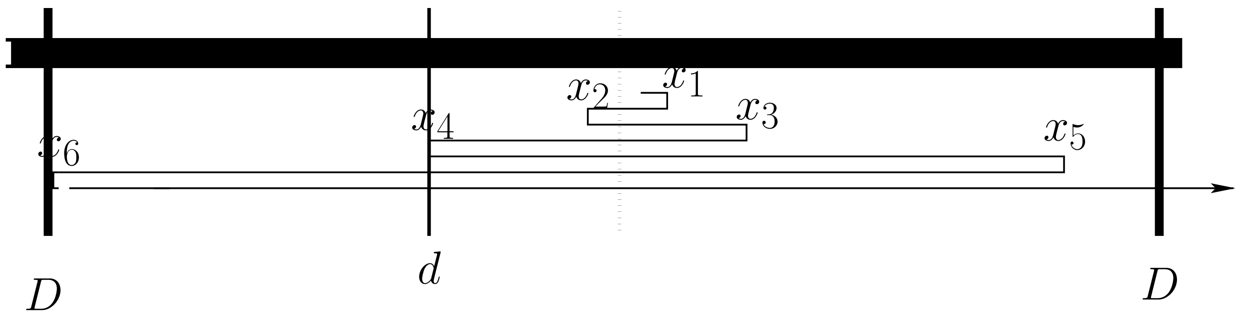
Other approach: Optimality for equations!

- Set: $\frac{\sum_{i=1}^{k+1} x'_i}{x'_k} = \frac{(C-1)}{2}$ for all k
- $\sum_{i=1}^{k+1} x'_i - \sum_{i=1}^{k} x'_i = \frac{(C-1)}{2} \left(x'_k - x'_{k-1} \right)$
- Thus: $C' \left(x'_k - x'_{k-1} \right) = x'_{k+1}$, Recurrence!
- Solve a recurrence! Analytically! Blackboard!
- Characteristical polynom: No solution $C' < 4$
- $x'_i = (i+1)2^i$ with $C' = 4$ is a solution! Blackboard! Optimal!



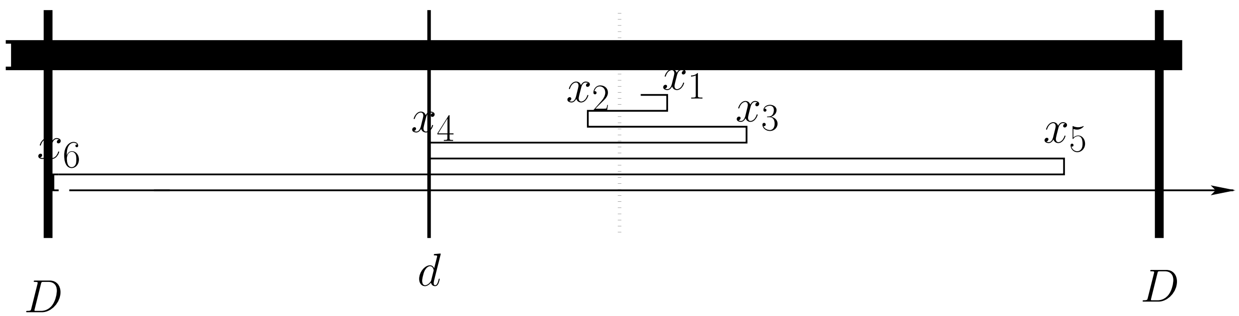
2-ray search, restricted distance

- Assume goal is no more than dist. $\leq D$ away
- Exactly D! Simple ratio 3!
- Find optimal startegy, minimize C!
- Vice-versa: C is given! Find the largest distance D (reach R) that still allows C competitive search.
- One side with $f_{\text{Ende}} = R$, the other side arbitrarily large!



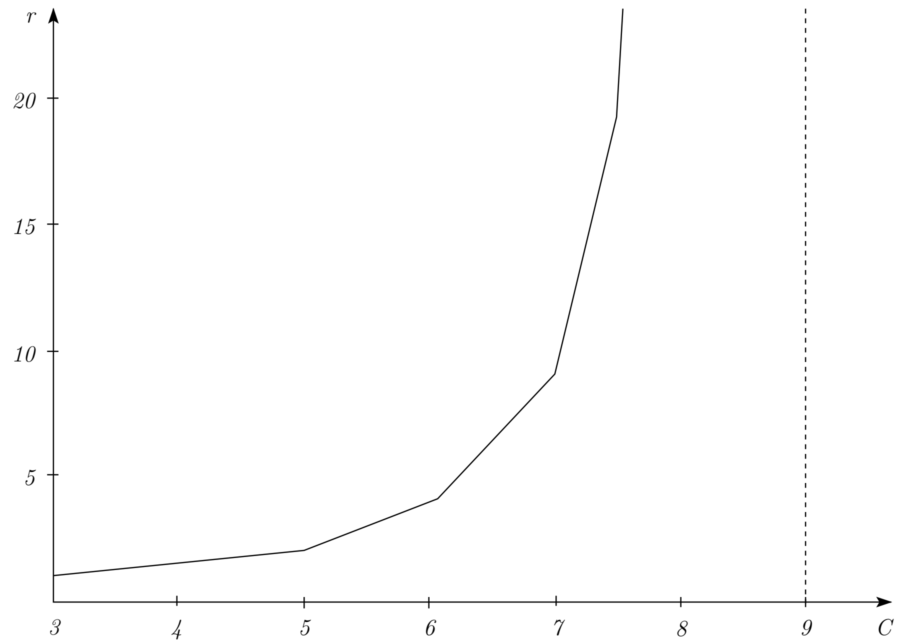
2-ray search, maximal reach R

- C given, optimal reach R!
- **Theorem** The strategy with equality in any step maximizes the reach R !
- Strategy: $\frac{\sum_{i=1}^{k+1} x_i}{x_k} = \frac{(C-1)}{2}$, first step: $x_1 = \frac{(C-1)}{2}$
- Recurrence: $x_0 = 1$, $x_{-1} = 0$, $x_{k+1} = \frac{(C-1)}{2}(x_k - x_{k-1})$
- Strategy is optimal! By means of the Comp. Geom. lecture!



2-ray search, maximal reach R

- $f(C) :=$ maximal reach depending on C
- Bends are more steps!



2-ray search, given distance R

- $f(C) :=$ maximal reach depending on C
- Rotate, R given, binary search!