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Problem Set 9

Problem 1

Suppose that we roll a standard fair dice seventeen times (independently). What is the probability that the sum is divisible by six? Use the principle of deferred decisions.

Problem 2

Let $X_1, \ldots, X_n$ be independent random variables with density functions $f_1, \ldots, f_n$ and let $\lambda_1, \ldots \lambda_n \in \mathbb{R}_+$ be arbitrary. Furthermore, let $f_i(x) \leq \phi$ for every $i \in \{1, \ldots, n\}$ and every $x \in \mathbb{R}$. Give an upper bound for the probability of $\lambda_1 X_1 + \ldots + \lambda_n X_n \in [a, a + \varepsilon]$, where $a \in \mathbb{R}$ and $\varepsilon > 0$ are fixed arbitrarily.

Problem 3

The knapsack problem (given n items, deterministic profits $p_1, \ldots, p_n \geq 0$, deterministic weights $w_1, \ldots, w_n > 0$ and a capacity W) can be solved by an algorithm with running time $\mathcal{O}(nP)$ where $P = \sum_{i=1}^{n} p_i$ is the sum of all profits. Explain the algorithm. What does the existence of this algorithm mean for the complexity of the knapsack problem?

Problem 4

Consider the following variant of the knapsack problem. Given n objects with ϕ-perturbed weights $w_1, \ldots, w_n \in [0, 1]$, a capacity W and profits $p_1, \ldots, p_n \in \mathbb{R}^{\geq 1}$, find a solution $x \in \{0, 1\}^n$ that maximizes the product

$$p(x) = \prod_{i:x_i=1} p_i$$

of the profits of the chosen items under the constraint that $w^t x \leq W$.

1. Can Theorem 6.13 be applied to this variant?
2. If so, does that imply that the problem has a polynomial smoothed complexity? Can you give an algorithm for the problem with polynomial smoothed complexity?