Discrete and Computational Geometry, WS1415
Exercise Sheet “2”: Randomized Algorithms for Geometric Structures II
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 28th of October, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 4: Triangulation (4 Points)

Given a set N of n points in the plane, a triangulation $T(N)$ of N is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to maintain the planarity. Let S_1, S_2, . . . , S_n be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $T(N)$ by computing $T(N^3)$, $T(N^4)$, ...,$T(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $T(N^{i+1})$ from $T(N^i)$ by adding S_{i+1}. (Hint: Add three dummy points, p_1, p_2, and p_3, in the infinity such that the outer boundary of $T(N^i \cup \{p_1, p_2, p_3\})$ is a triangle whose vertices are p_1, p_2, and p_3 for $1 \leq i \leq n$.)

1. Describe the insertion of S_{i+1}
2. Define a conflict relation between a triangle in $T(N^i)$ (i.e., $T(N^i \cup \{p_1, p_2, p_3\})$) and a point in $N \setminus N^i$
3. Prove the expected cost of inserting S_{i+1} to be $O(\frac{n}{i+1})$ and the expected cost of construction $T(N)$ to be $O(n \log n)$

Exercise 5: Planar Convex Hull by Conflict Lists (4 Points)

Given a set N of n points in the plane, a convex hull $H(N)$ of N is a minimal convex polygon containing N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}.

1. Describe the insertion of S_{i+1}
2. Define a conflict relation between an edge of $H(N^i)$ and a point in $N \setminus N^i$
3. Prove the expected cost of inserting S^{i+1} to be $O(\frac{n}{i+1})$ and the expected cost of construction $H(N)$ to be $O(n \log n)$.

Exercise 6: Voronoi Diagrams (4 Points)

Given a set S of n points in the Euclidean plane, the Voronoi diagram $V(S)$ partitions the plane into Voronoi regions $\mathrm{VR}(p, S)$, $p \in S$, such that all points in $\mathrm{VR}(p, S)$ share the same nearest site p among S. We make a general position assumption that no more than three points of S are located on the same circle. Let e, v, and u be the numbers of edges, vertices, unbounded faces of $V(S)$.

1. Please prove $e = 3(n-1) - u$ and $v = 2(n-1) - u$. (Hint: use Euler's forumla)
2. Please explain that if u is fixed, the number of vertices will not increase without the general position assumption.