Discrete and Computational Geometry, SS 14
Exercise Sheet "7": Minkowskis Theorem and Applications
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday June 3rd, 14:00 pm**. *There will be a letterbox in the LBH building, close to Room E01.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 19: Proof details Two-Squares-Theorem (4 Points)

1. For $p = 17$, present the corresponding values of q, a and b, i and j in the proof of the Two-Squares-Theorem (Theorem 11). Finally $p = a^2 + b^2$ for $a, b \in \mathbb{Z}$ has to be fulfilled.
2. Prove the following statement: For the factor ring $\mathbb{Z}_p$ for a prime p only $a = \overline{1}$ and $a = -\overline{1}$ gives a solution for $a^2 = \overline{1}$.

Exercise 20: Application of Minkowskis Theorem (4 Points)

Consider the regular (5×5) lattice around the origin. Calculate the required expansion (radius r) of the *trees* at the lattice points so that any line $Y = aX$ hits at least one of the *trees*. Do the calculation in the following ways:

1. Calculate the radius r directly and precisely by considering the corresponding circles and lines.
   (W.l.o.g. only two cases have to be considered!)

2. Make use of the Minkowski Theroem and compute a non-trivial radius r that fulfills the requirement.

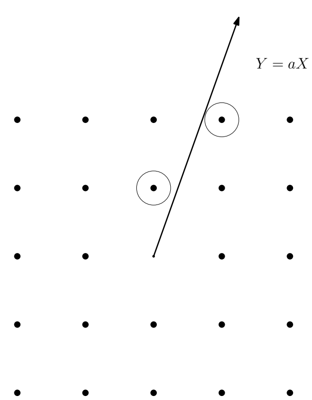

Figure 1: The regular (5×5) grid. The line passes the circles.

Exercise 21: Extension of Minkowskis Theorem (4 Points)

Prove the following statement for the standard lattice:

If $C \subset \mathbb{R}^d$ is convex, symmetric around the origin, bounded and for the volume $vol(C) > k2^d$ holds, then C contains at least $2k$ lattice points.