Discrete and Computational Geometry, SS 14
Exercise Sheet "9": Spanners and WSPDs
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday July 1st, 14:00 pm**. *There will be a letterbox in the LBH building, close to Room E01.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

**Exercise 28: Spanners and Closest Pairs (4 Points)**

Let $S$ denote a finite point set in $\mathbb{R}^d$. Let $1 < t \leq 2$ and let $G = (S, E)$ be a $t$-spanner with verex set $S$ and edge set $E$.

**a)** Show that for at least one closest pair $v, w$ in $S$ the edge $\{v, w\}$ belongs to $E$. Furthermore, if $t < 2$, this is even true for all closest pairs.

**b)** Let $p$ be a nearest neighbor of $q$ in $S$. Does this imply that $\{p, q\}$ belongs to $E$?

**Exercise 29: WSPD and Centers (4 Points)**

Prove or disprove the following statement: Two point sets $A$, $B$ with bounding box $R(A)$ and $R(B)$ are well-separated with parameter $s$, if and only if there are two circles $C_A$ und $C_B$ of some radius $r$, where $R(A) \subset C_A$, $R(B) \subset C_B$ and the distance between $C_A$ and $C_B$ is $\geq r \cdot s$, *and the center of $C_A$ and of $C_B$ coincides with the center of the bounding box of $A$ and of $B$, respectively.*

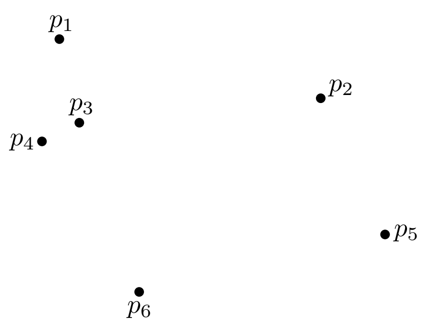
**Exercise 30:** **WSPD 2-dimensional Example** **(4 Points)**

Consider the point set $S \subset \mathbb{R}^2$ depicted twice below. Use the algorithm presented in the lecture to construct a WSPD of $S$, given the separation ratio $s = 1$.

Start with computing the split-tree, and draw the resulting bounding boxes.

Use these bounding boxes to construct the WSPD. You may assume that the procedure FindPairs($v$,$w$) only verifies if the two point sets $S_v$ and $S_w$ are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box.

1)

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