Discrete and Computational Geometry, SS 14
Exercise Sheet "10": Spanners and WSPDs
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday July 1st, 14:00 pm**. *There will be a letterbox in the LBH building, close to Room E01.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 28: Spanners and Closest Pairs (4 Points)

Let S denote a finite point set in $\mathbb{R}^d$. Let $1 < t \leq 2$ and let $G = (S, E)$ be a t-spanner with verex set S and edge set E.

a) Show that for at least one closest pair v, w in S the edge $\{v, w\}$ belongs to E. Furthermore, if $t < 2$, this is even true for all closest pairs.

b) Let p be a nearest neighbor of q in S. Does this imply that $\{p, q\}$ belongs to E?

Exercise 29: WSPD and Centers (4 Points)

Prove or disprove the following statement: Two point sets A, B with bounding box $R(A)$ and $R(B)$ are well-separated with parameter s, if and only if there are two circles C_A und C_B of some radius r, where $R(A) \subset C_A$, $R(B) \subset C_B$ and the distance between C_A and C_B is $\geq r \cdot s$, *and the center of C_A and of C_B coincides with the center of the bounding box of A and of B, respectively.*

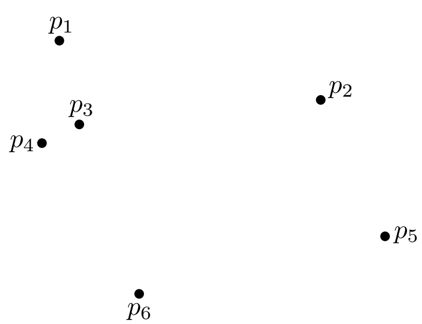
Exercise 30: WSPD 2-dimensional Example (4 Points)

Consider the point set $S \subset \mathbb{R}^2$ depicted twice below. Use the algorithm presented in the lecture to construct a WSPD of S, given the separation ratio $s = 1$.

Start with computing the split-tree, and draw the resulting bounding boxes.

Use these bounding boxes to construct the WSPD. You may assume that the procedure FindPairs(v,w) only verifies if the two point sets S_v and S_w are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box.

1)



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