Geometric Dilation of Closed Planar Curves: A New Lower Bound

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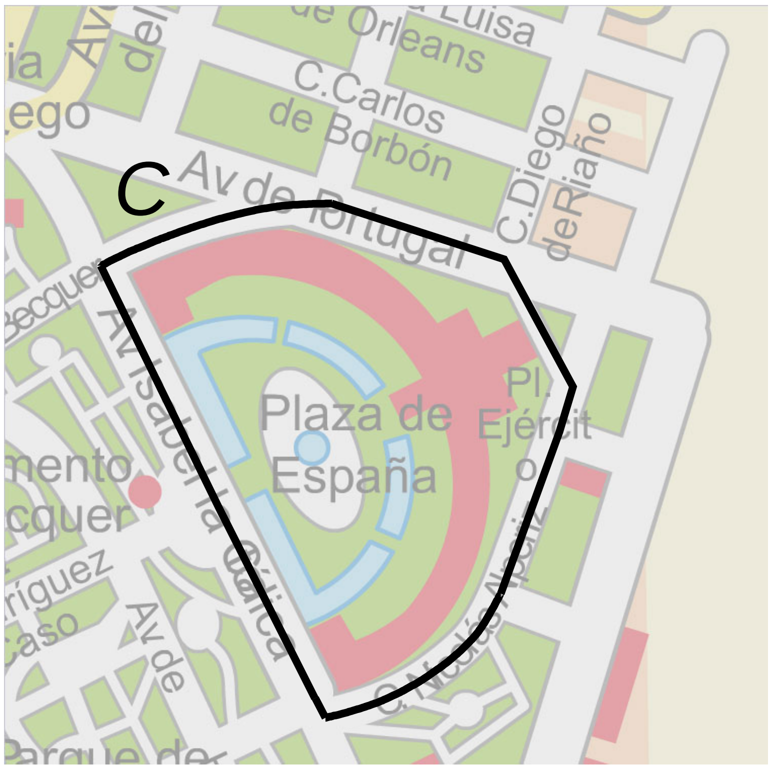
Outline

- Geometric Dilation
- Motivation
- Non-Convex Cycles
- Partition Pairs
- Breadth Measures
- Central Symmetrization
- Lower Bound
- Results

Geometric Dilation

- for embedded planar graph G
- $p, q \in C$
- $d_C(p, q) :=$ length of shortest path on C connecting p, q
- $|pq| =$ Euclidean distance
- *Dilation*
  $\delta_C(p, q) := d_C(p, q) / |pq|$
- *Geometric Dilation*
  $$\delta(C) := \sup_{p,q \in C, p \neq q} \delta_C(p, q)$$

Geometric Dilation



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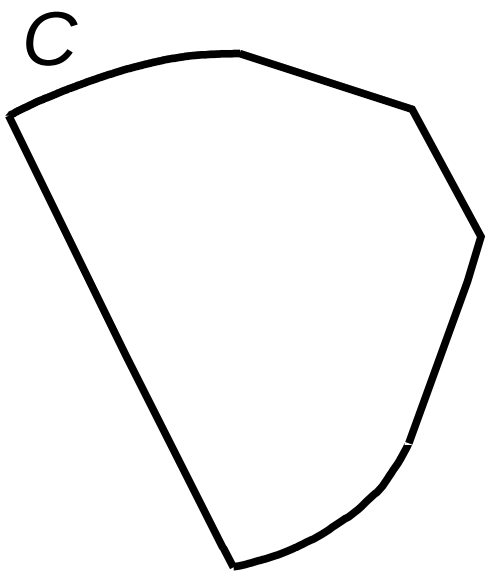

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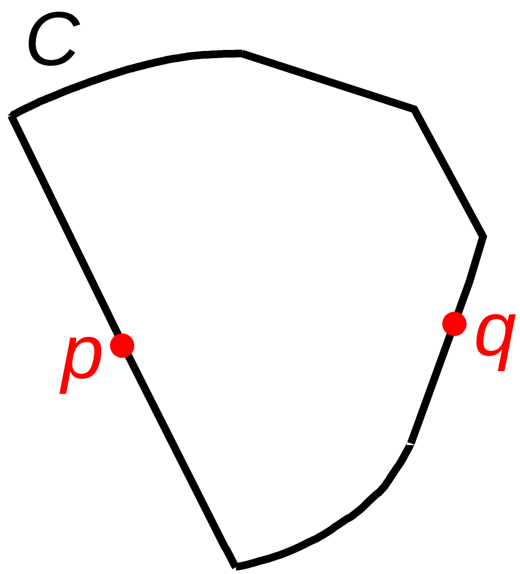

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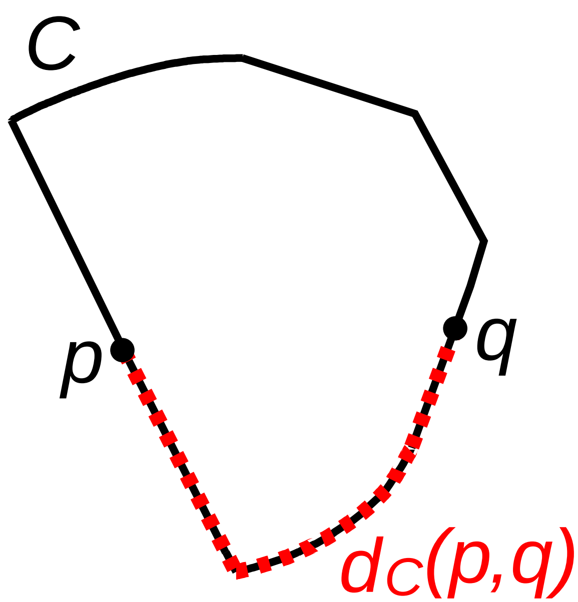

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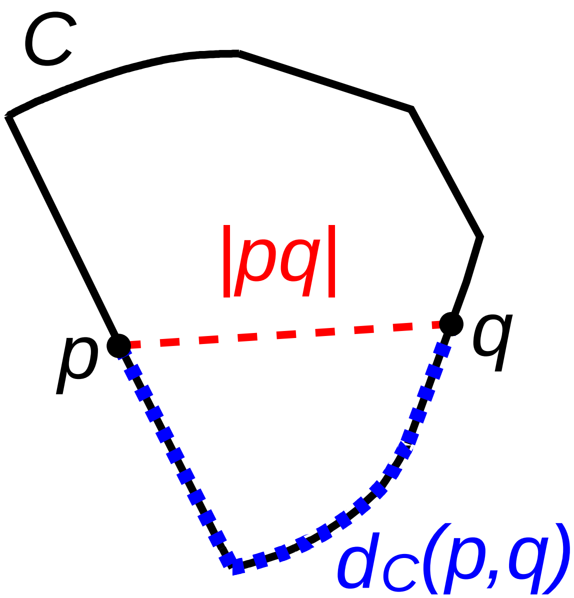

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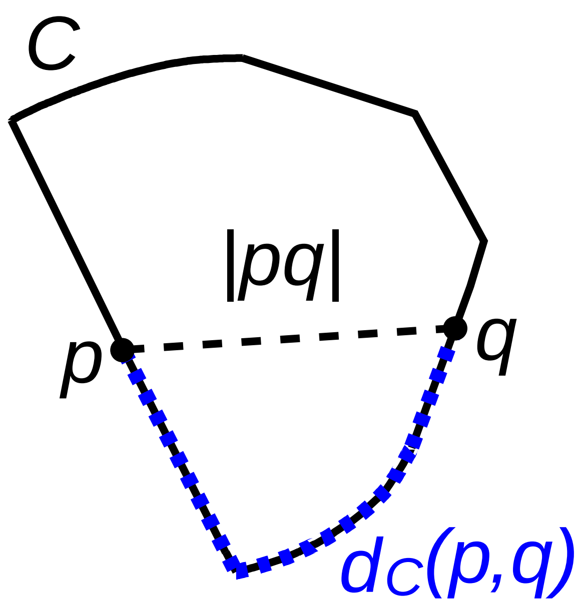

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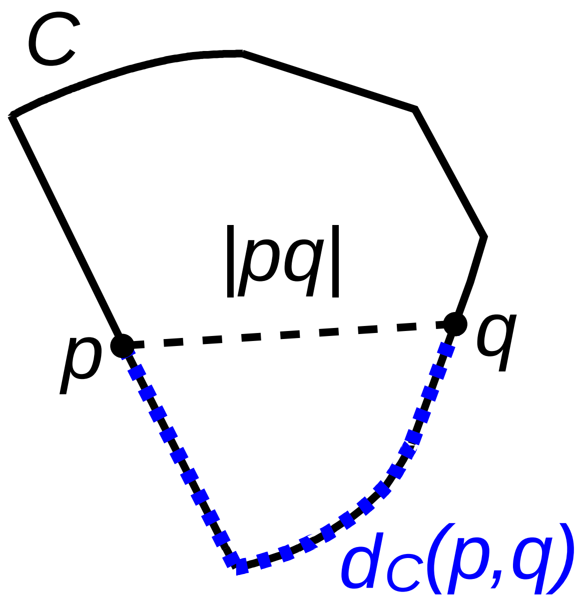

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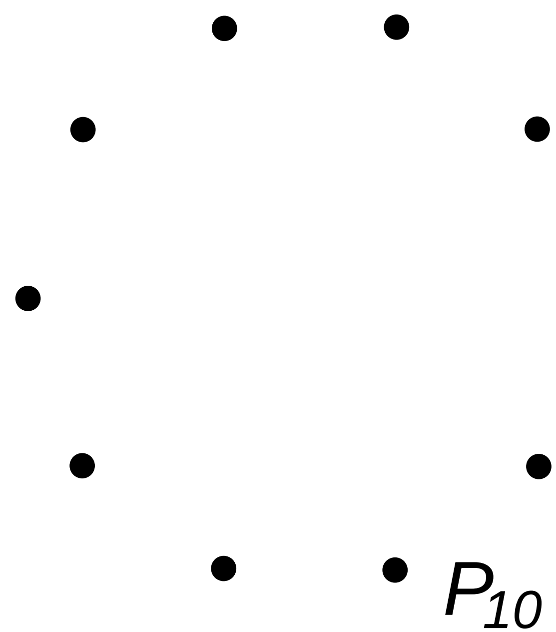

- ISAAC'03: point set P_{10} :
  $\forall$ graph G embedding P_{10}:
  $\delta(G) \geq \frac{\pi}{2}$

- most important step:
  $\forall$ cycle C: $\delta(C) \geq \frac{\pi}{2}$

- Which cycles (besides circles) attain $\delta(C) = \frac{\pi}{2}$?

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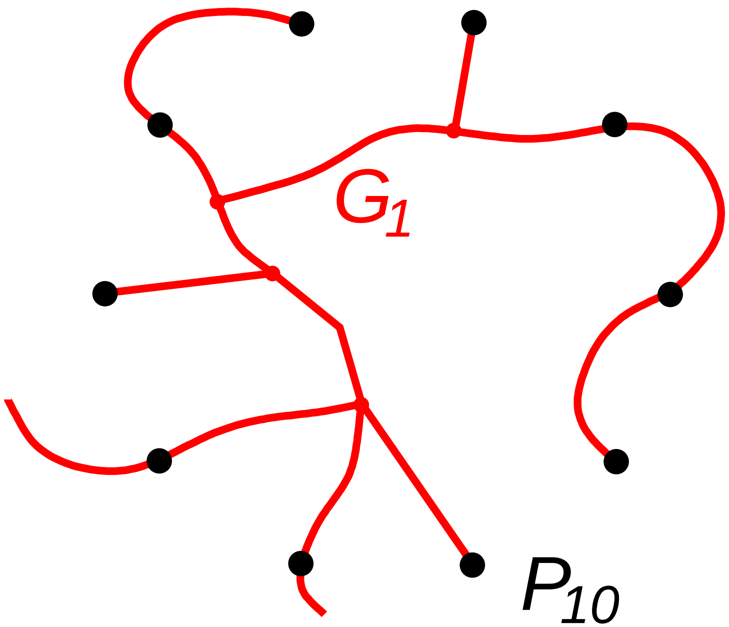

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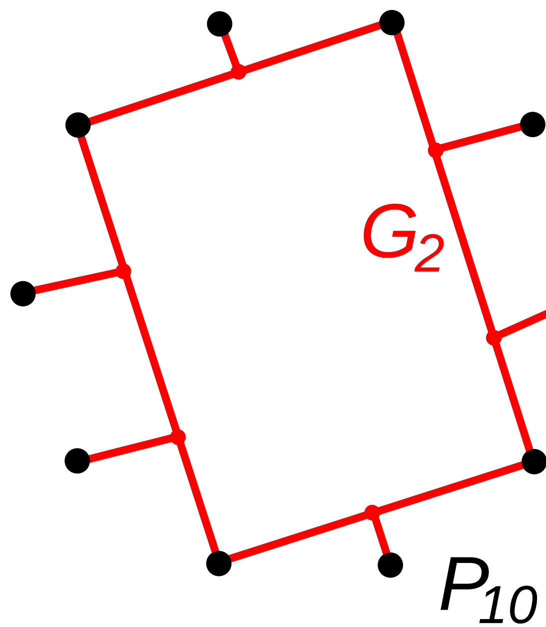

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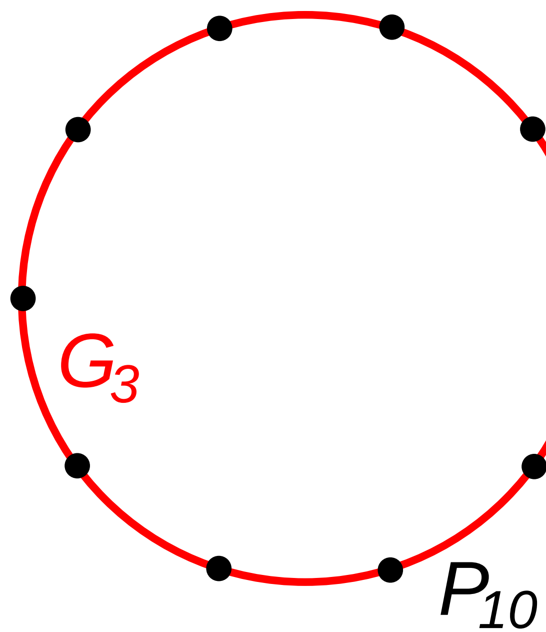

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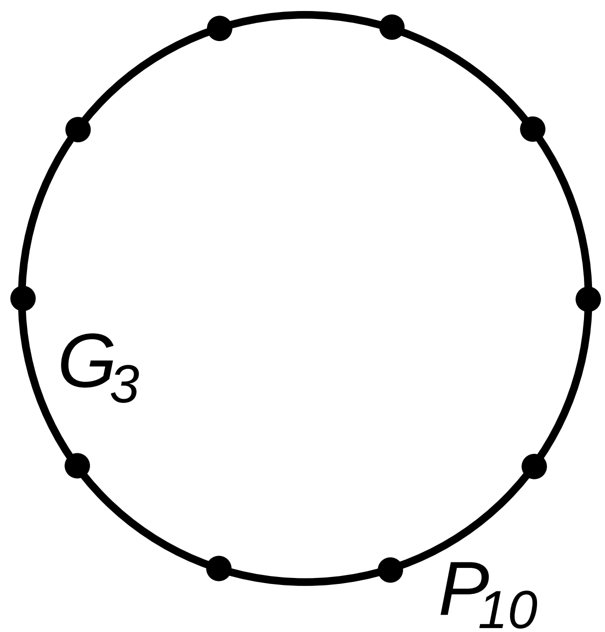

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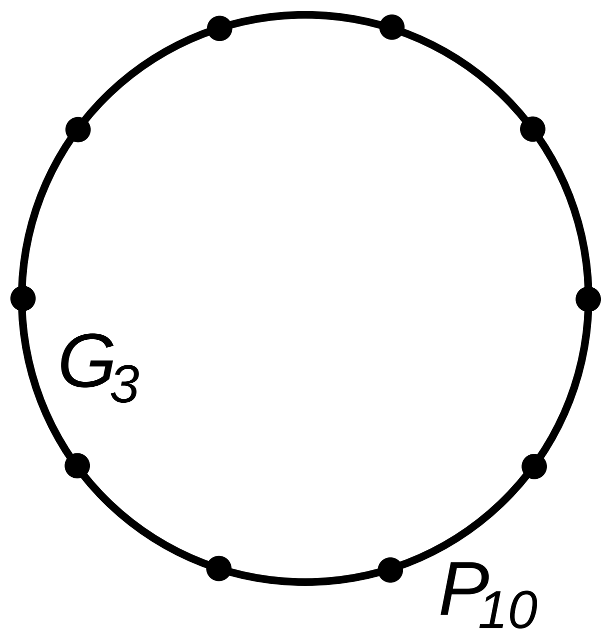

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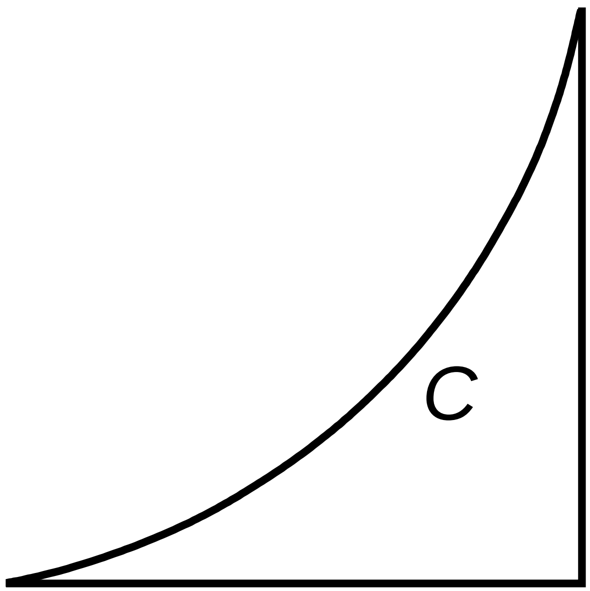
Non-Convex Cycles



- C cycle
- $\mathrm{ch}(C)$ = convex hull
- Then: $\delta(\mathrm{ch}(C)) \leq \delta(C)$
- from now on:
  CONVEX CYCLES!

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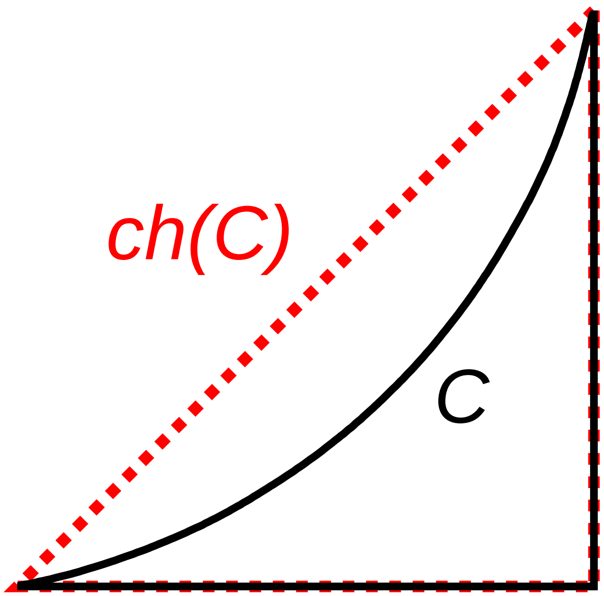

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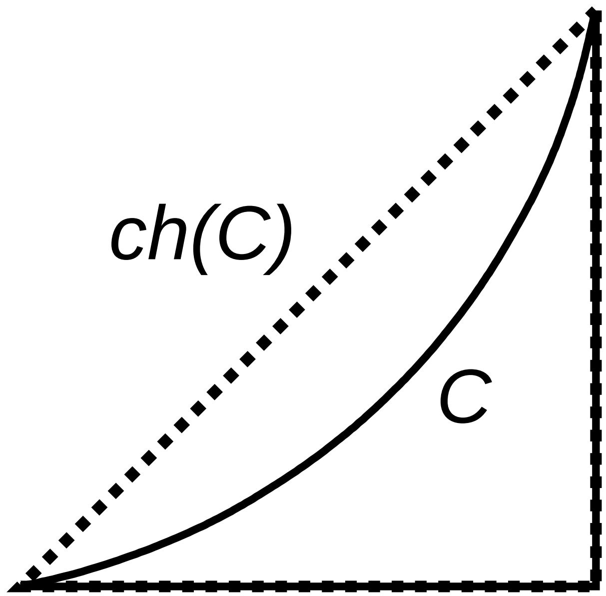

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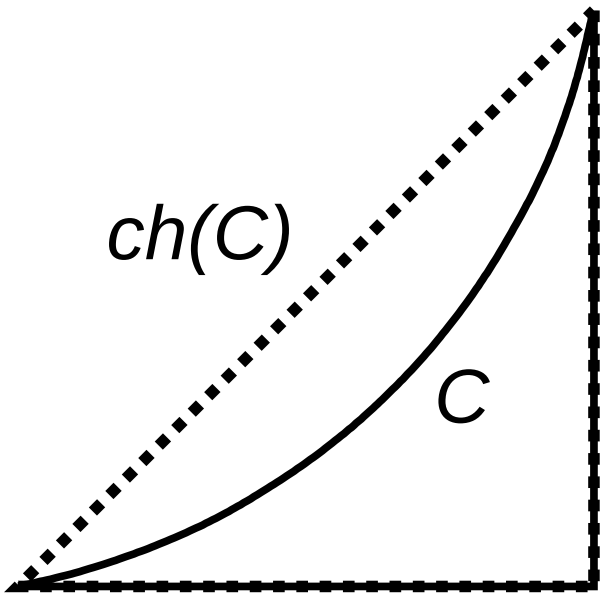

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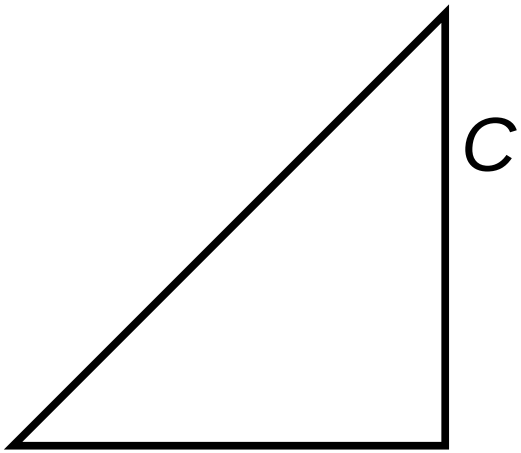
Partition Pairs



- $\delta(C) = \sup \; d_C(p,q) / |pq|$
- max. candidates: $d_C(p,q)$ maximal
- $\Rightarrow$ *Partition Pair:* $d_C(p,\tilde{p}) = \frac{|C|}{2}$
- indeed: a partition pair attains maximum dilation for convex C (follows from Ebbers-Baumann et al., '01)
- $\Rightarrow \delta(C) = \frac{|C|}{2h(C)}$,
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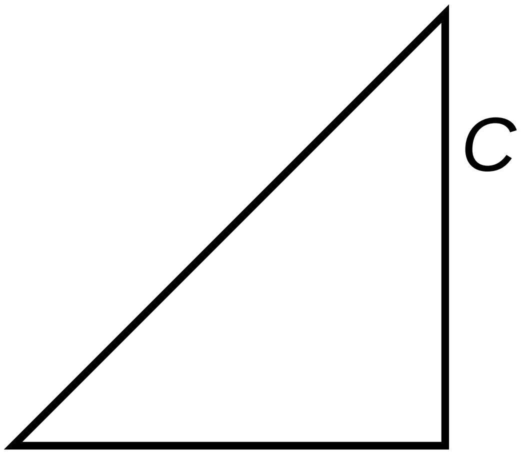

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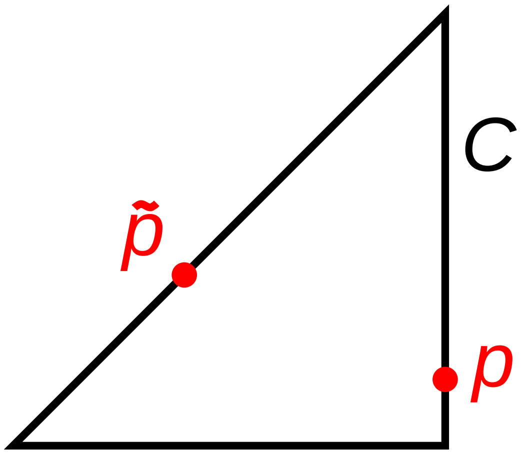

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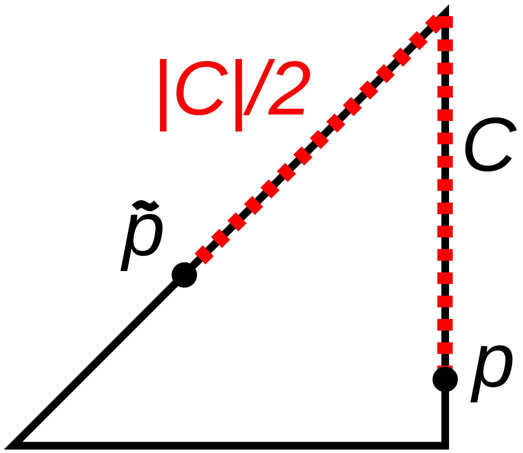

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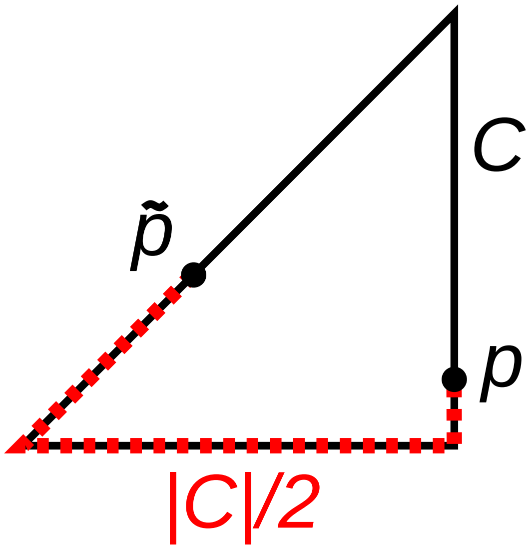

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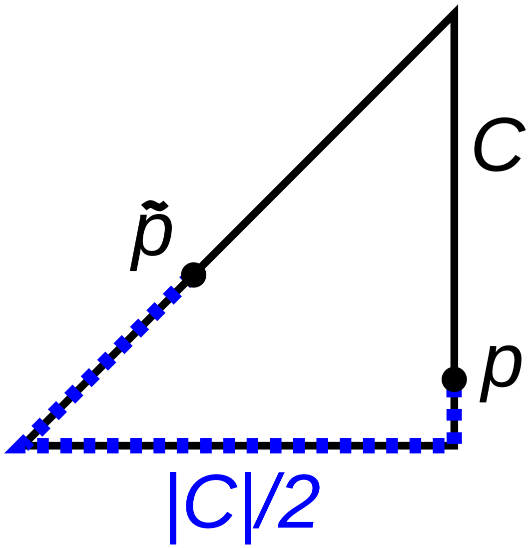

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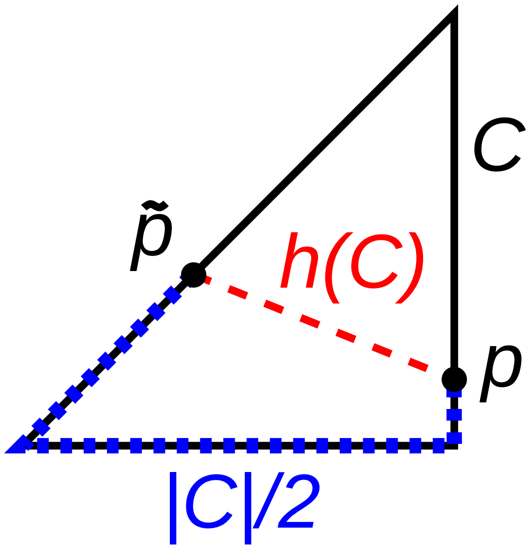

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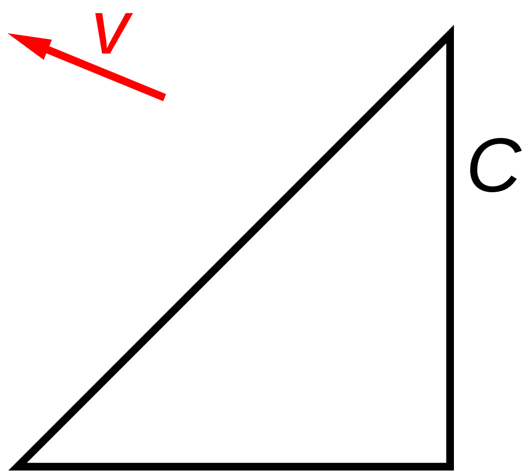

- direction $v \in \mathbb{S}^1$ (unit vector)
- *Partition Pair Distance*
  $h_C(v) := |p\tilde{p}|$,
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- *Length* $l_C(v)$:= length of longest stick with dir. v fitting into C
- *Breadth* $b_C(v)$:= distance of calipers orthogonal to v
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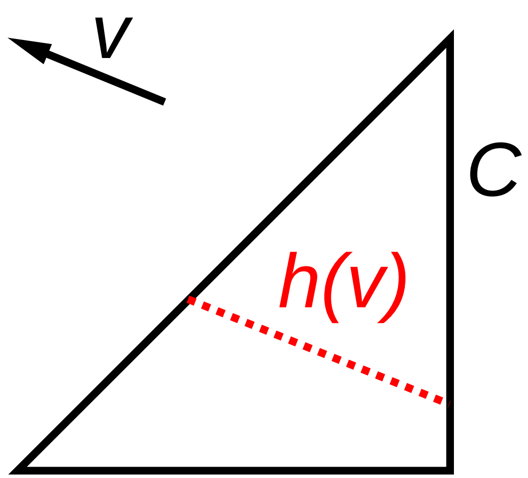

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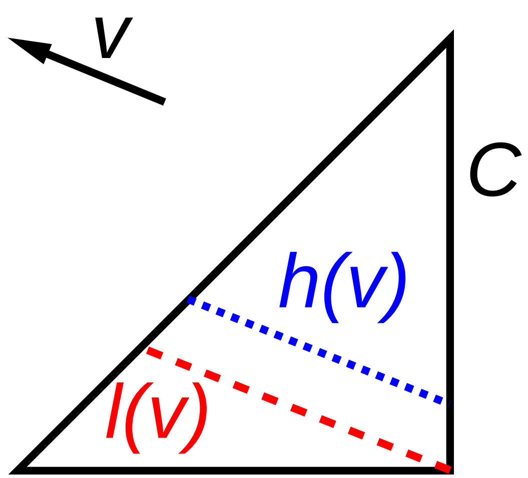

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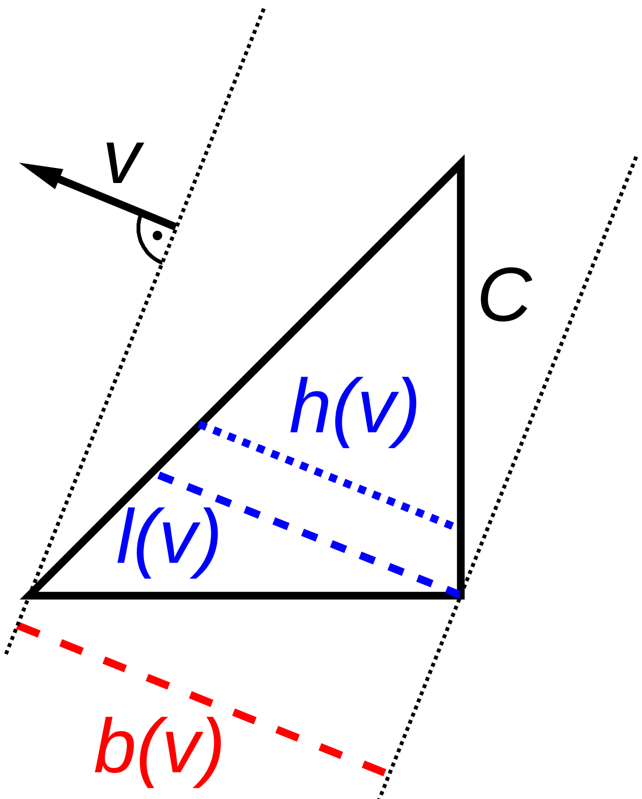

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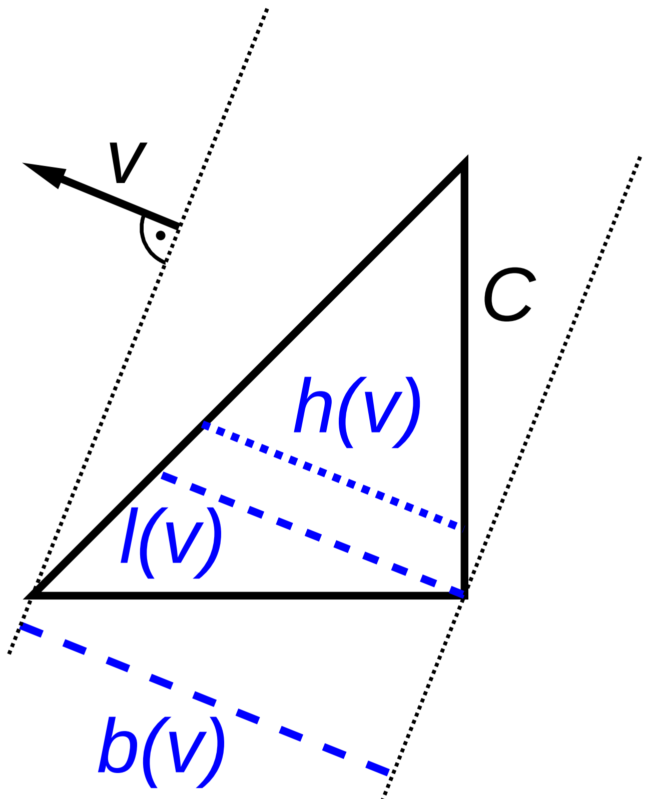

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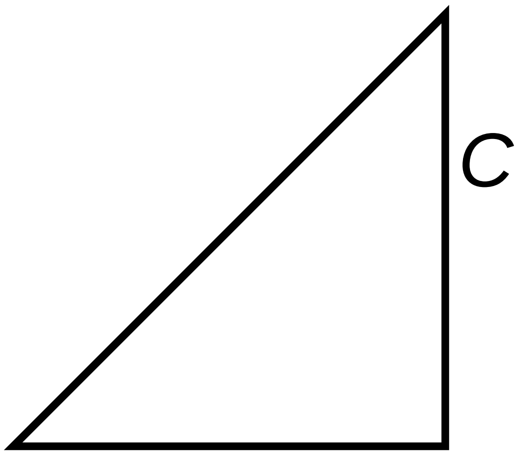

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- *Minimum Partition Pair Distance*
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* see e.g. Gritzmann, Klee '92

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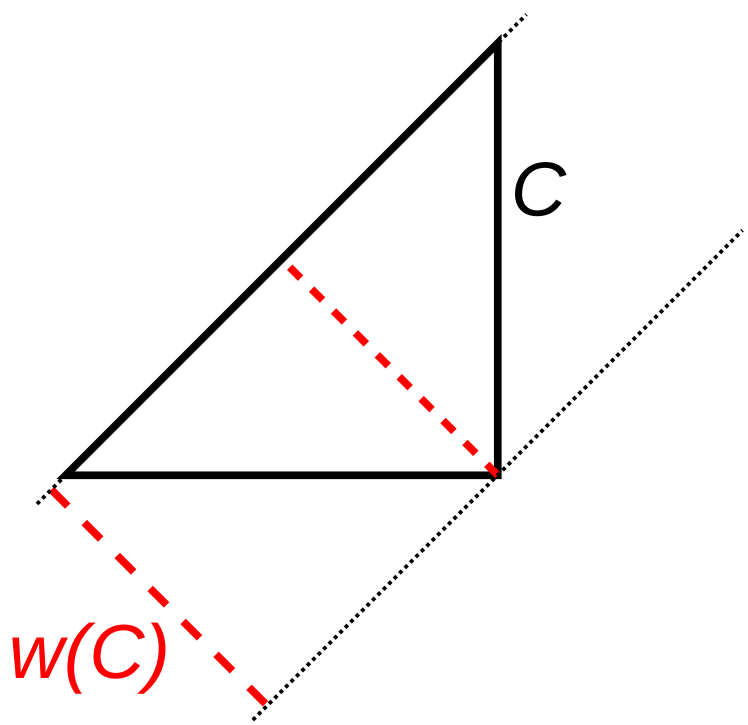

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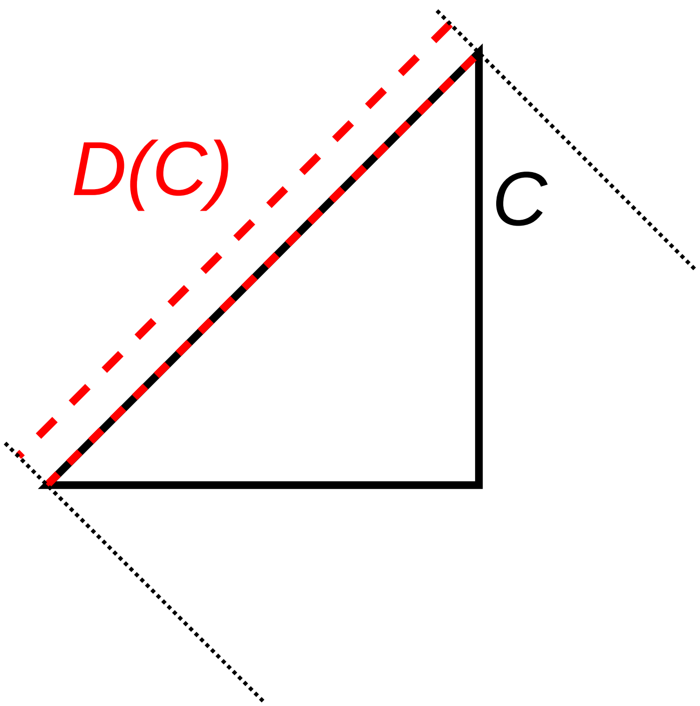

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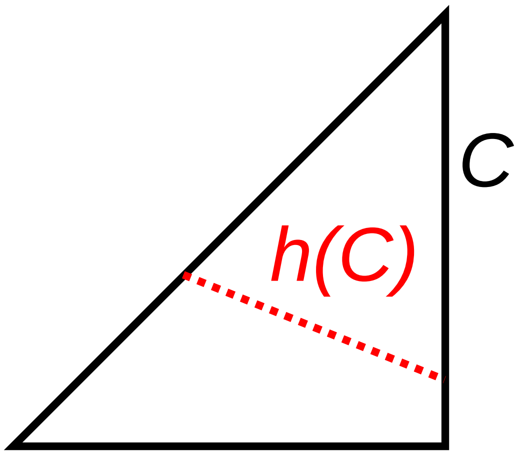

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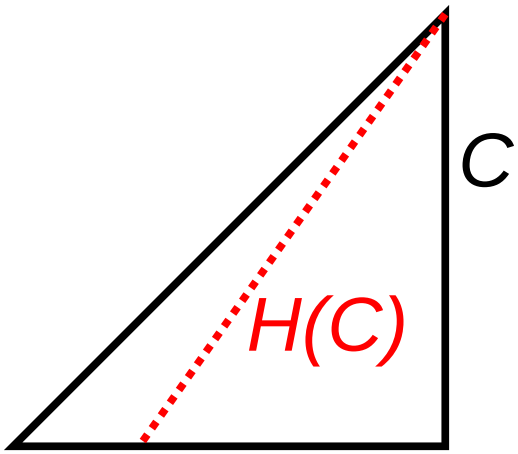

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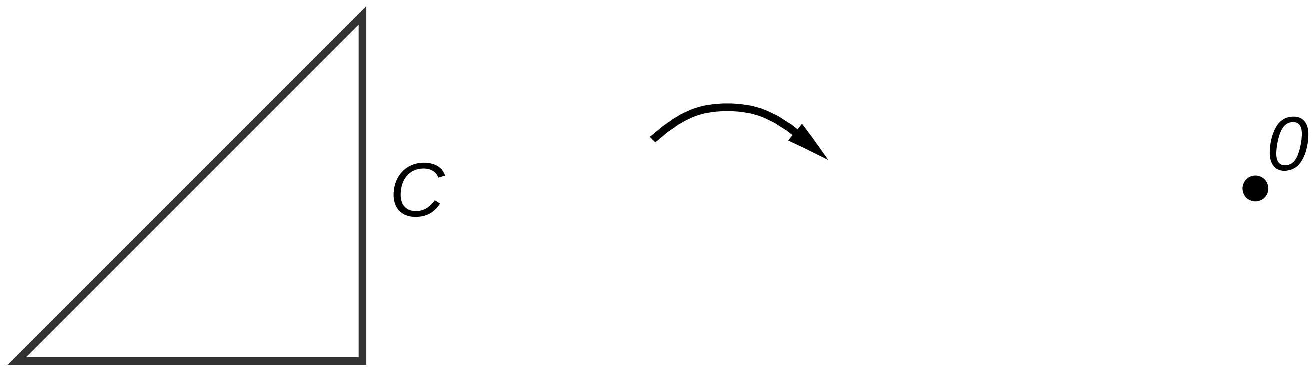
Central Symmetrization



- move centers of pairs attaining $l_C(v)$ (longest stick) to origin

- new cycle $=: C'$

(see e.g. Eggleston '58)

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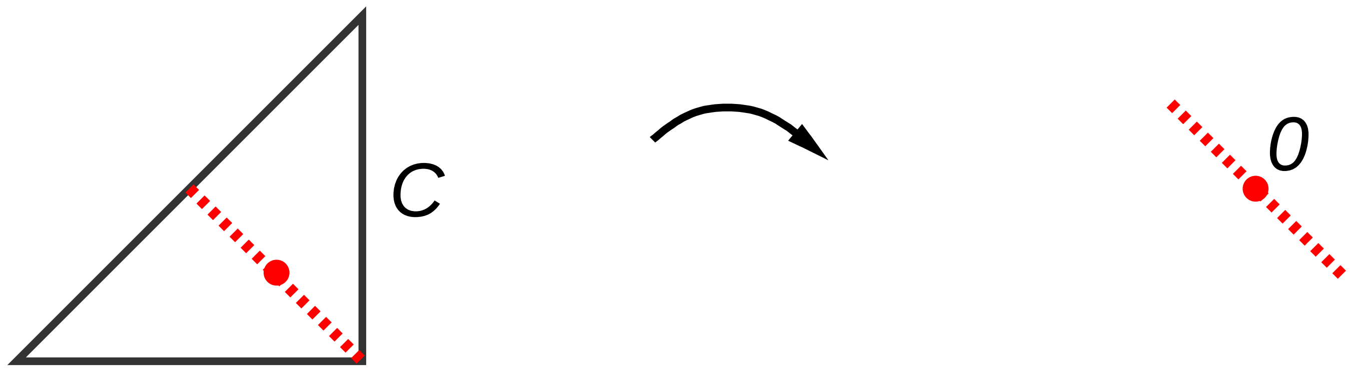

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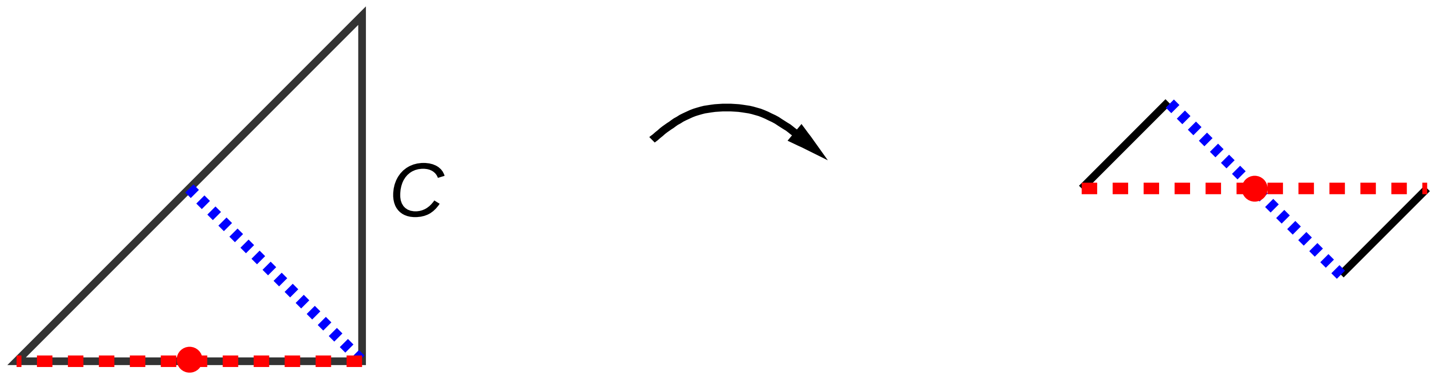

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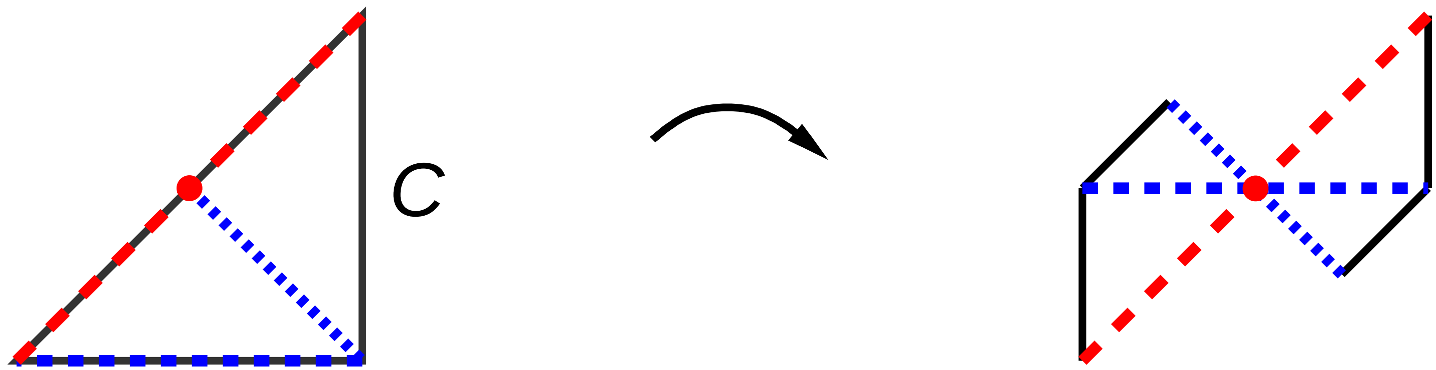

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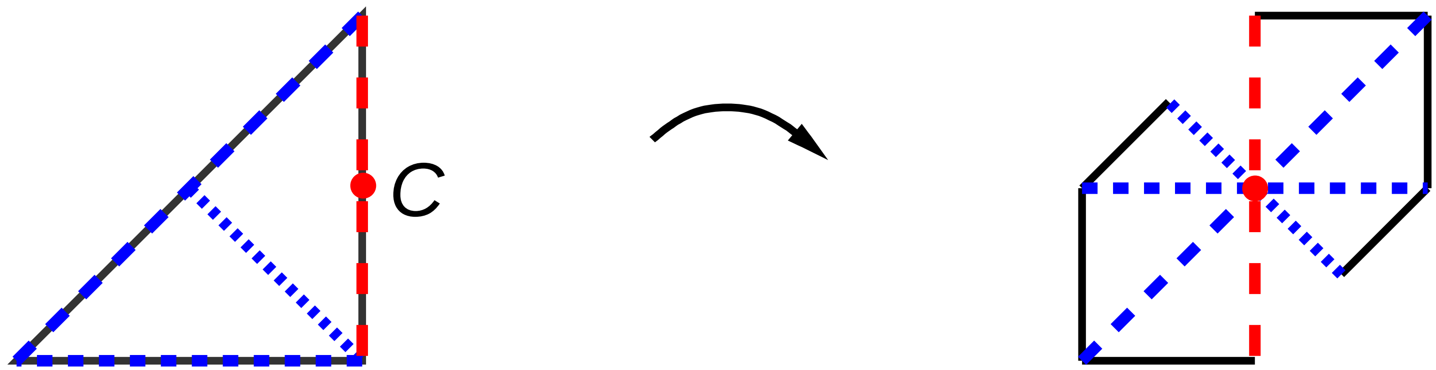

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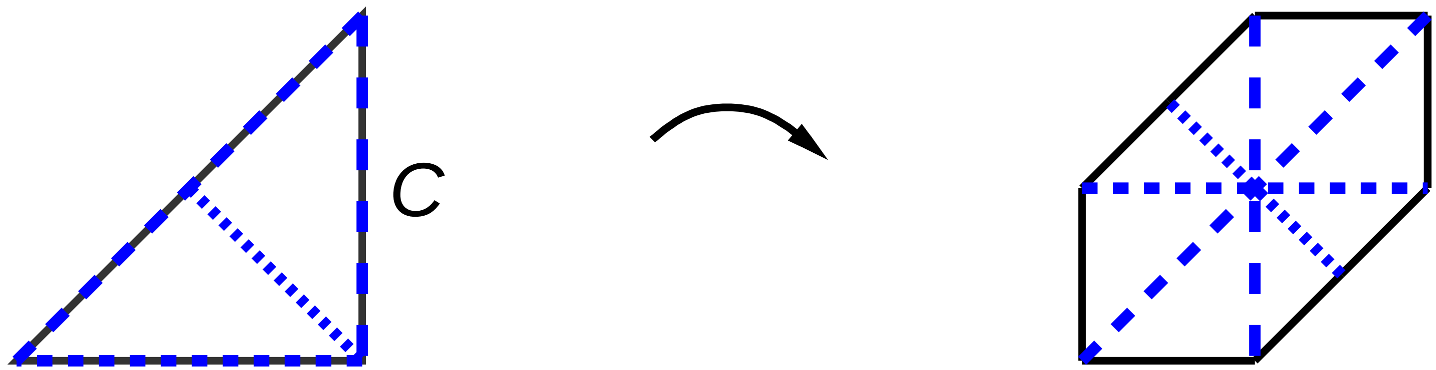

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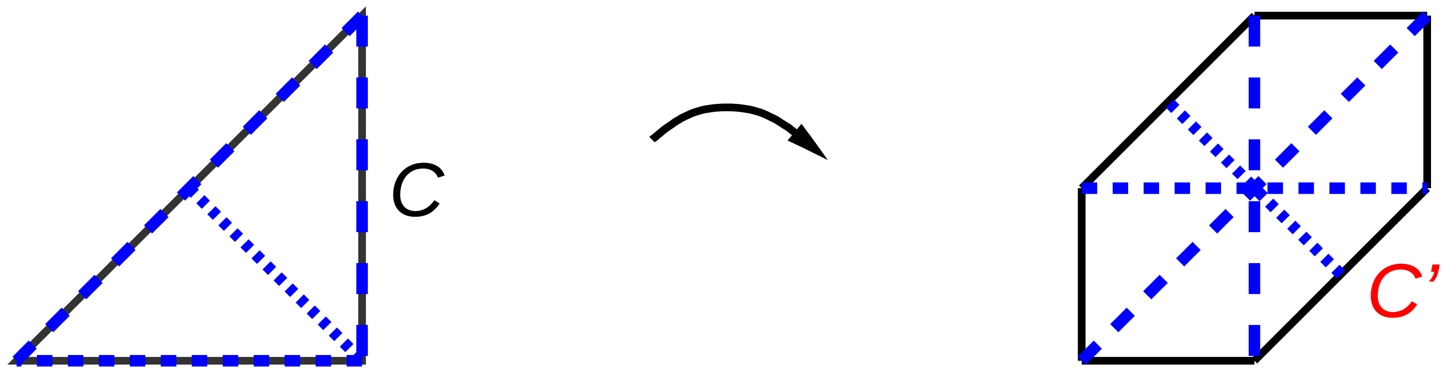

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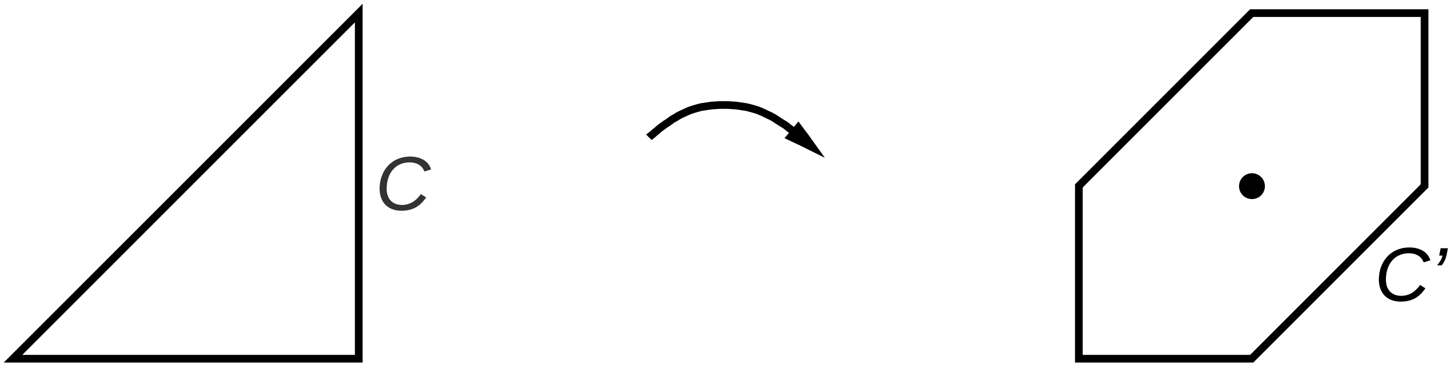
Properties Central Symmetrization



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b) $l_{C'}(v) = l_C(v)$

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d) $w(C') = w(C)$, $D(C') = D(C)$

e) $|C'| = |C|$

f) $h_{C'}(v) \geq h_C(v)$

g) $\delta(C') \leq \delta(C)$

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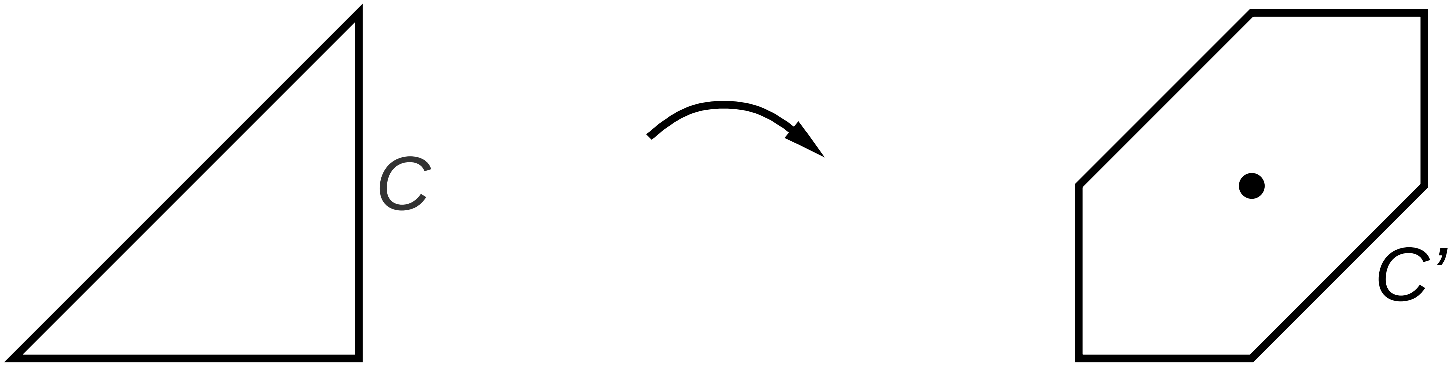

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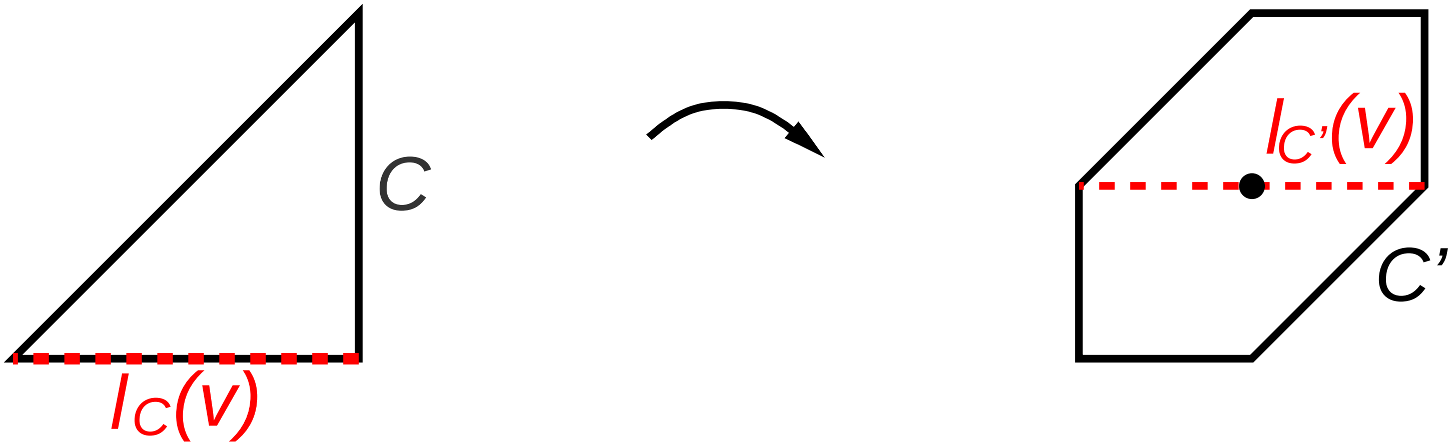

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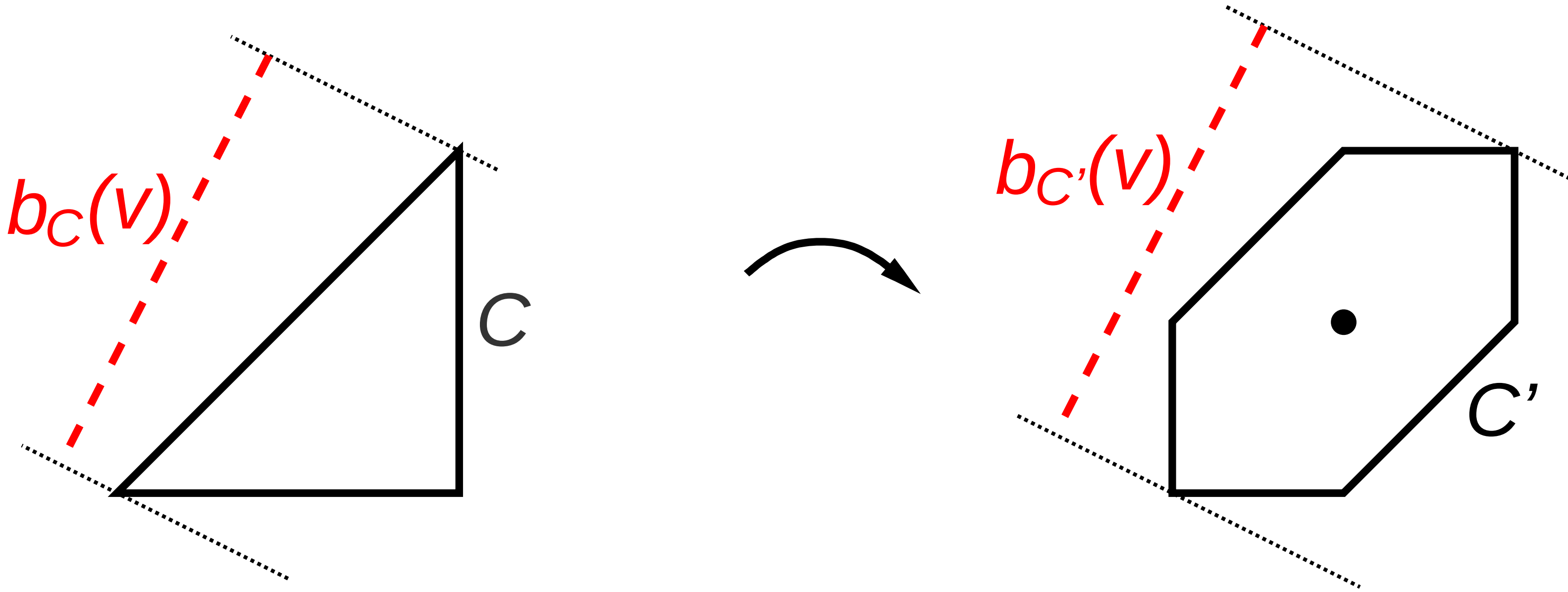

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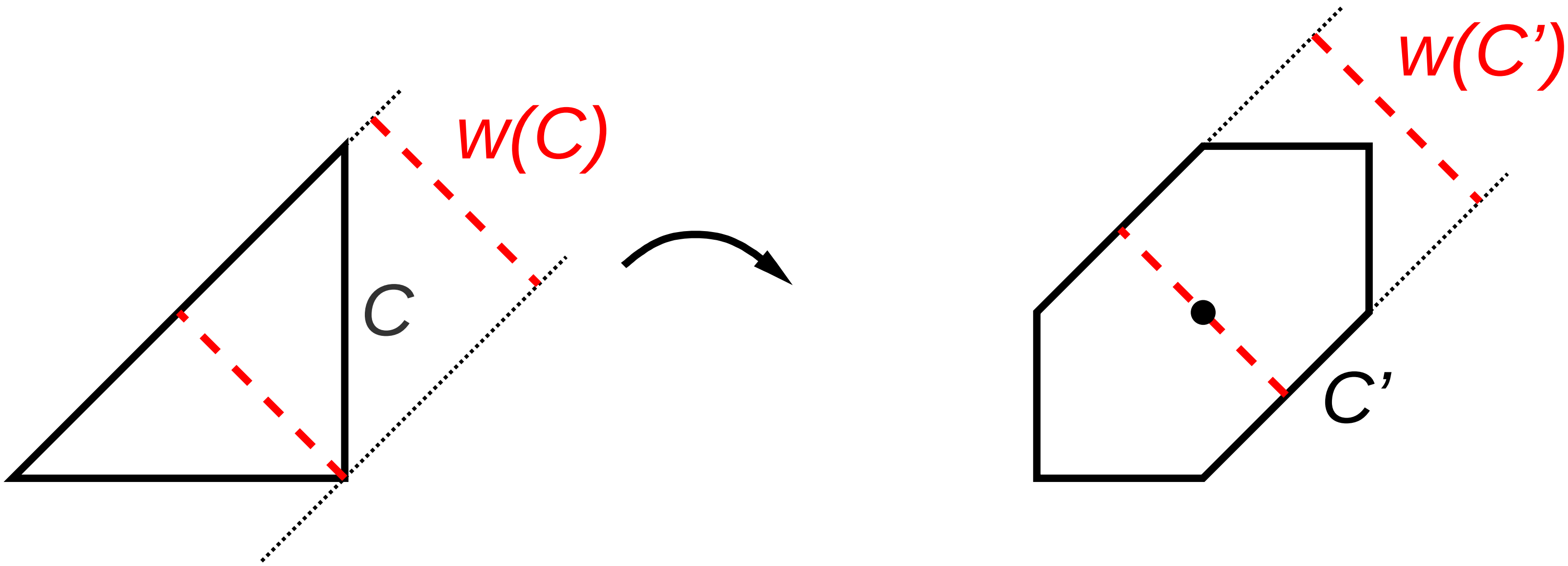

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(follows immediately from b) or c))

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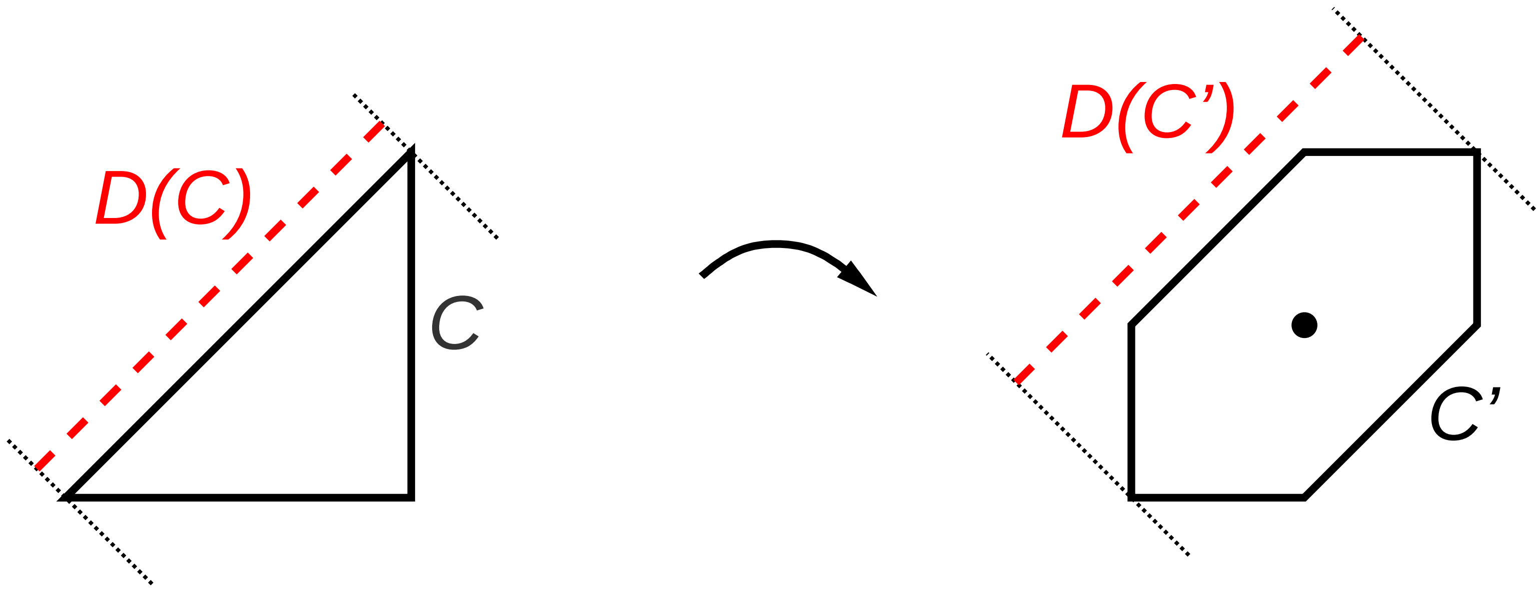

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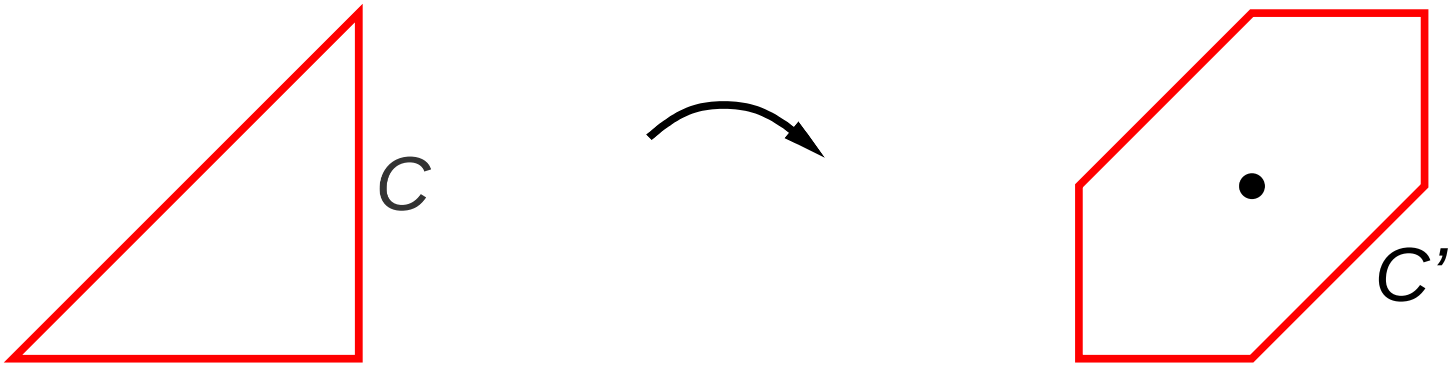

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(follows from c) and Cauchy's Surface Area Formula)

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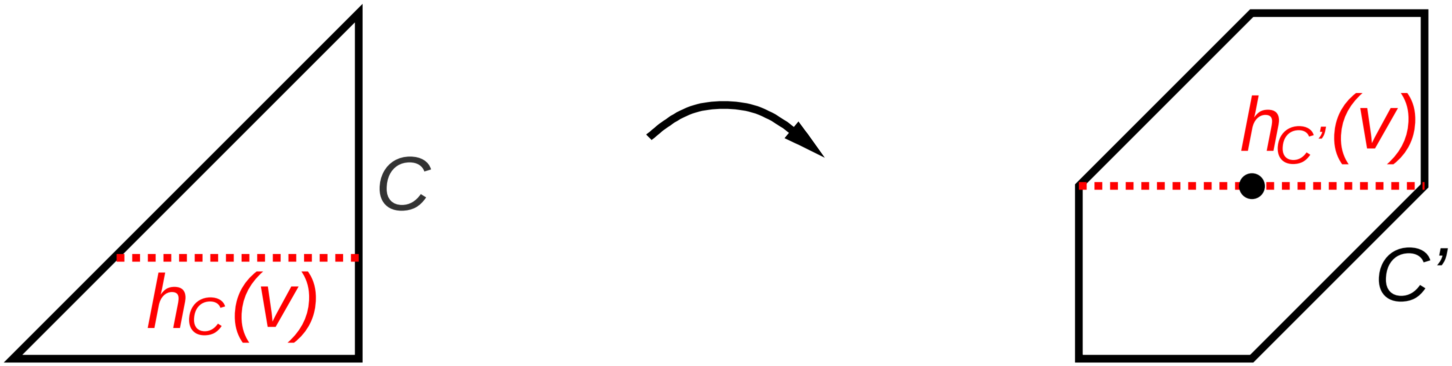

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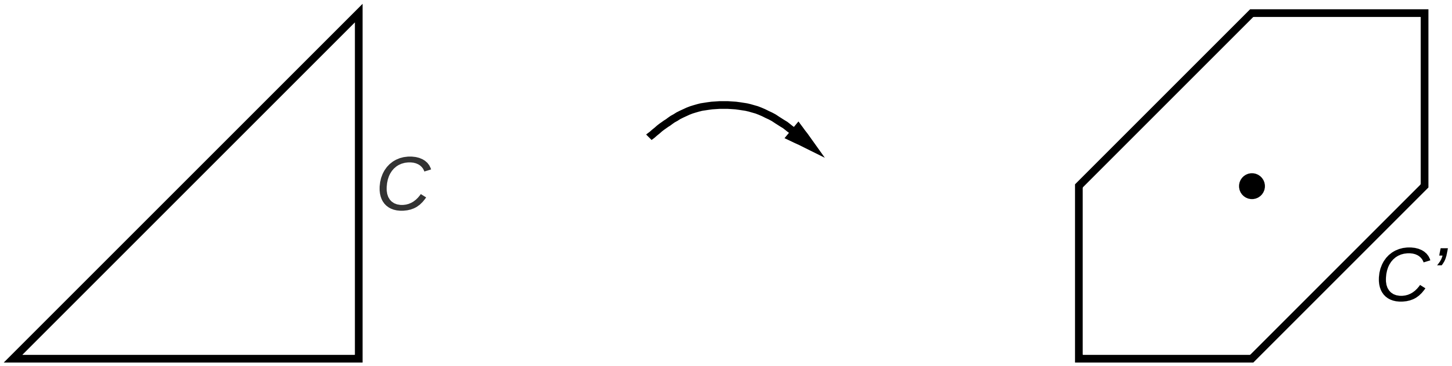
$$\left(h_{C'}(v) \overset{\text{point-sym.}}{=} l_{C'}(v) \overset{\text{b)}}{=} l_C(v) \geq h_C(v) \right)$$

Properties Central Symmetrization



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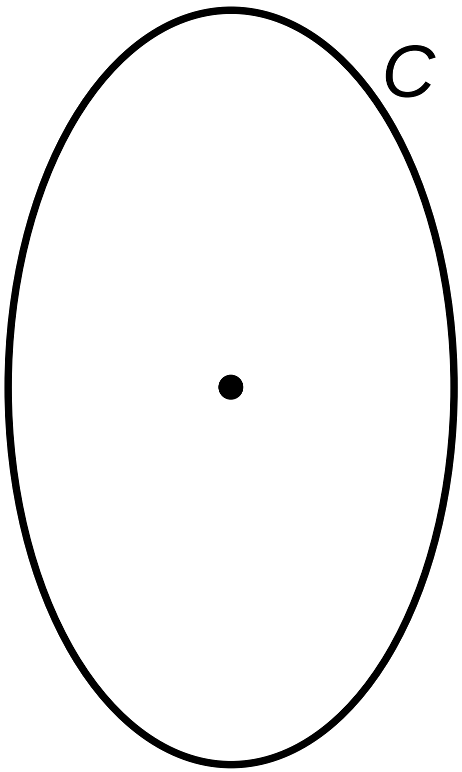
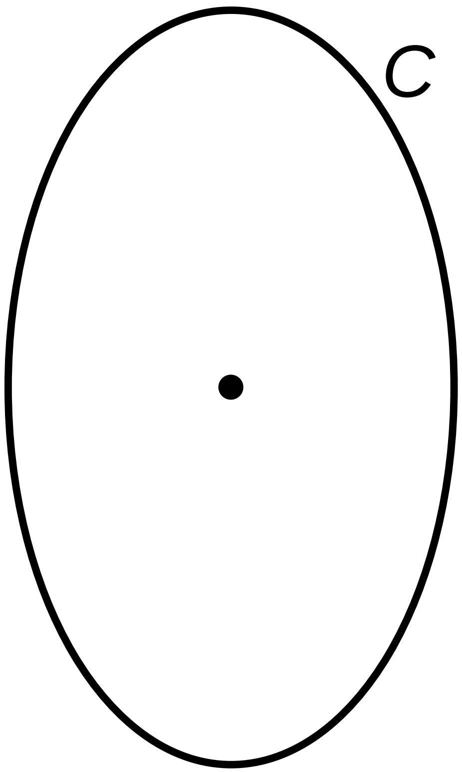
$$\left(\delta(C') = \frac{|C'|}{2h(C')} \overset{\text{e), f)}}{\leq} \frac{|C|}{2h(C)} = \delta(C) \right)$$

Lower Bound



- C point-symmetric (about origin)
  $\Rightarrow$ partition pairs $(p, -p)$,
  $h_C(v) = l_C(v)$, $w = h$, $D = H$
- C does not enter $B_{w/2}(0)$
- $\exists$ partition pair $(q, -q)$ of distance $H(C) = D(C)$
- $\Rightarrow \delta(C) = \frac{|C|}{2h} \geq \frac{|\tilde{C}|}{2h}$
  $$= \arcsin\left(\frac{w}{D}\right) + \sqrt{\left(\frac{D}{w}\right)^2 - 1}$$

Lower Bound



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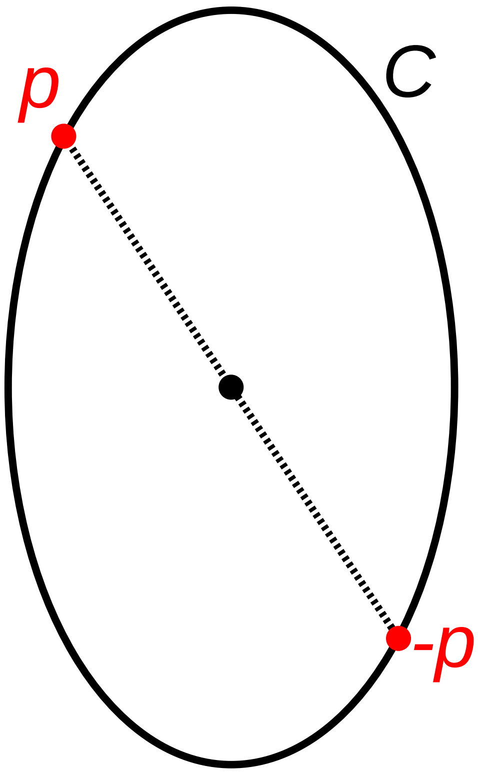

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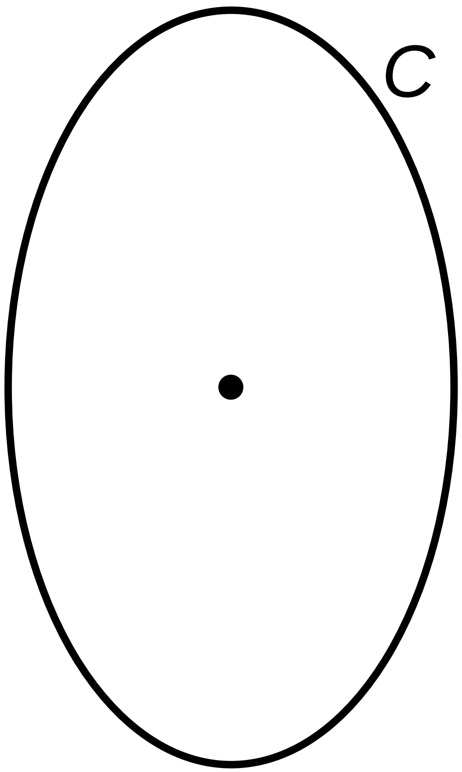

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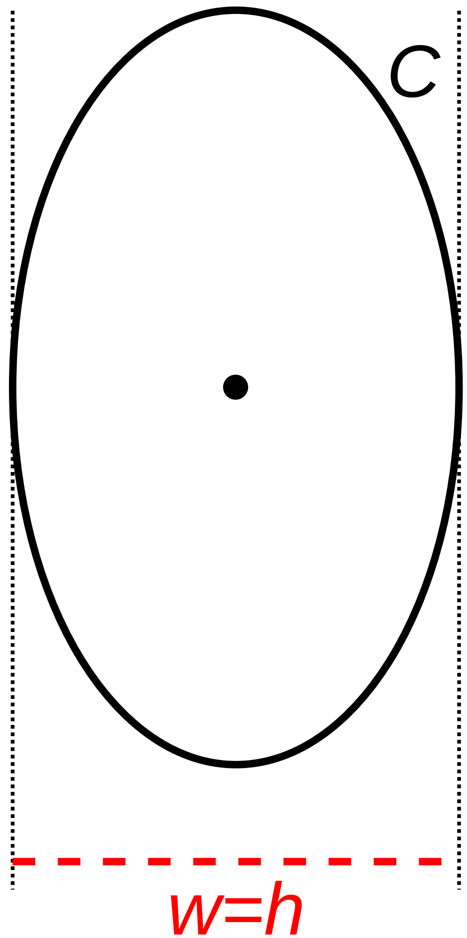

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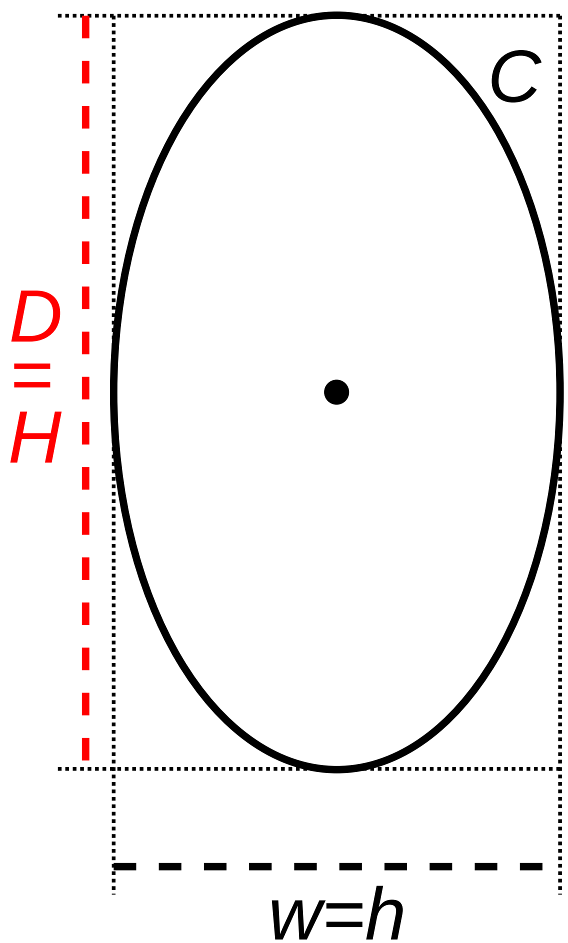

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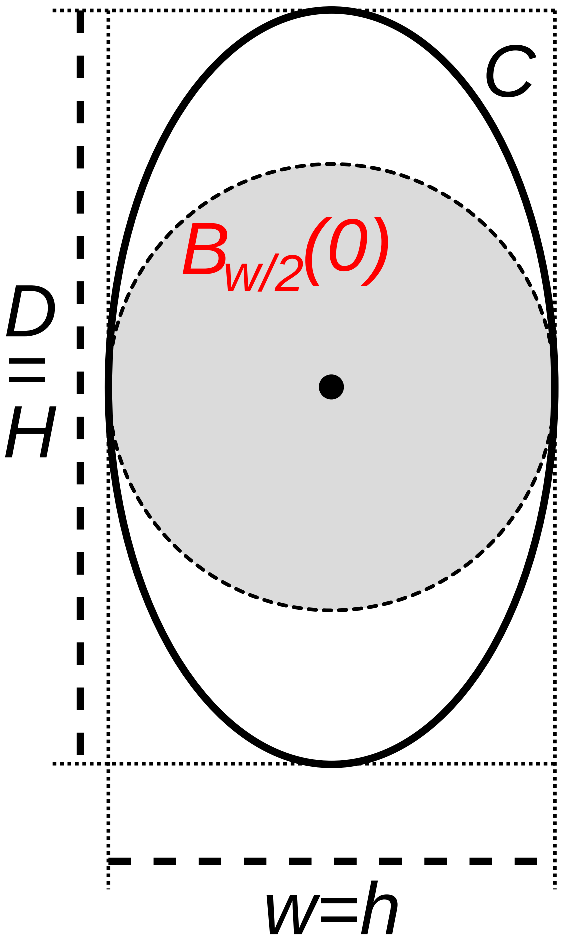

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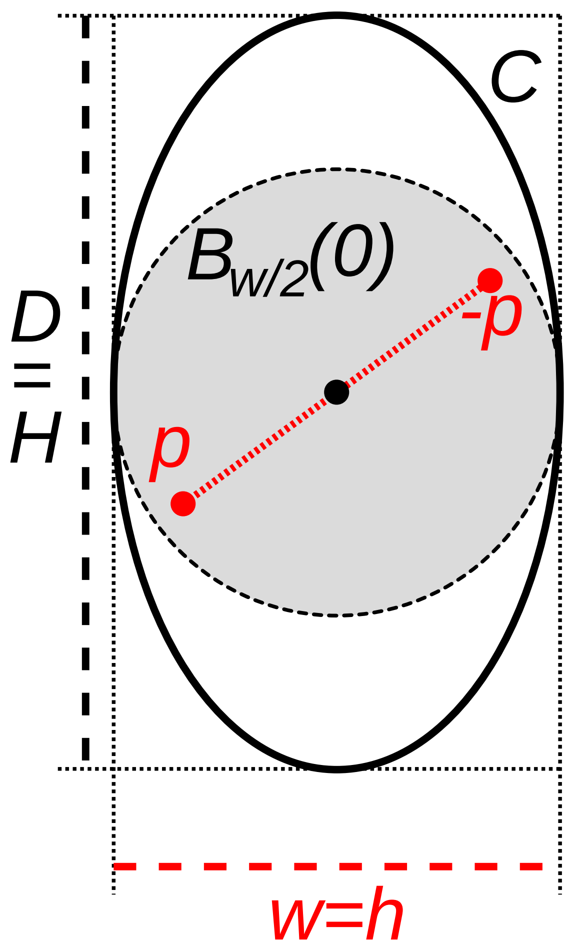

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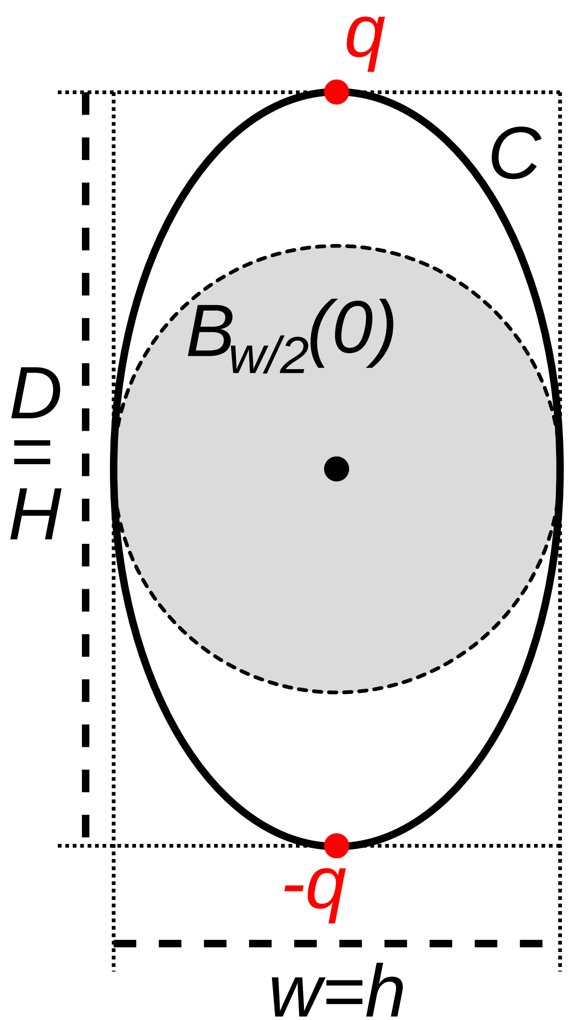

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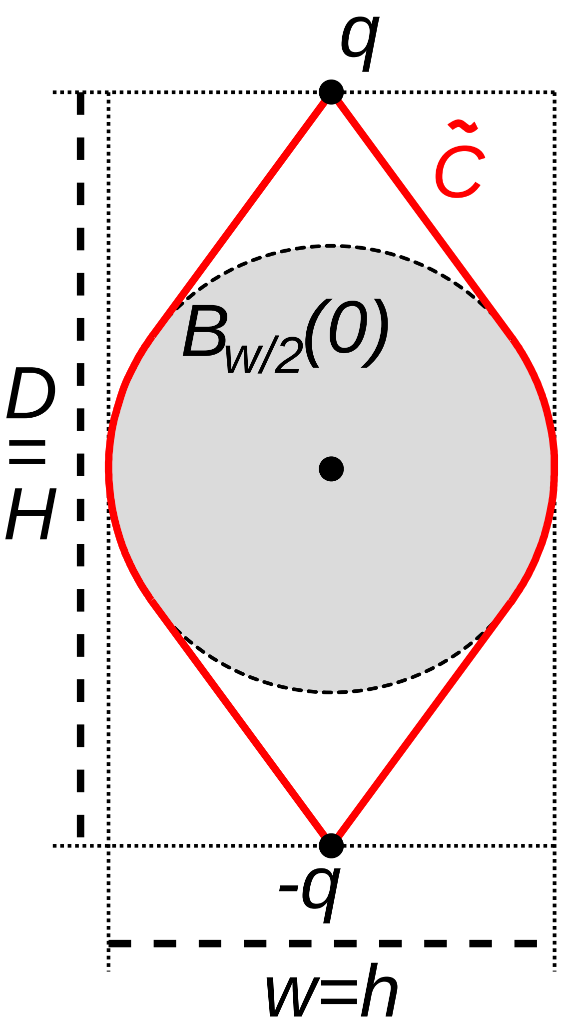

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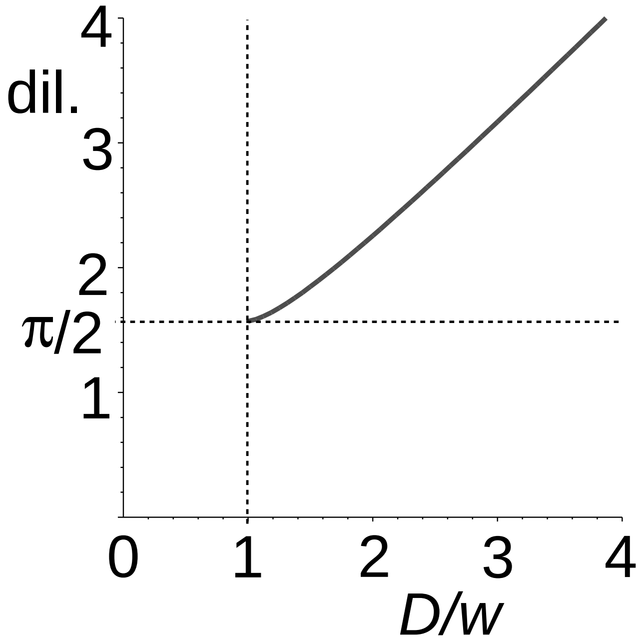
Results



$$\arcsin\left(\frac{w}{D}\right) + \sqrt{\left(\frac{D}{w}\right)^2 - 1}$$

- lower dilation bound extends to arbitrary convex cycles by central symmetrization
- $\delta(C) = \pi/2 \Rightarrow w = D$ (cycle of constant breadth)
- Partition Pair Transformation (analogously moves partition pairs to origin): $\delta(C) = \pi/2 \Rightarrow C$ point-symmetric
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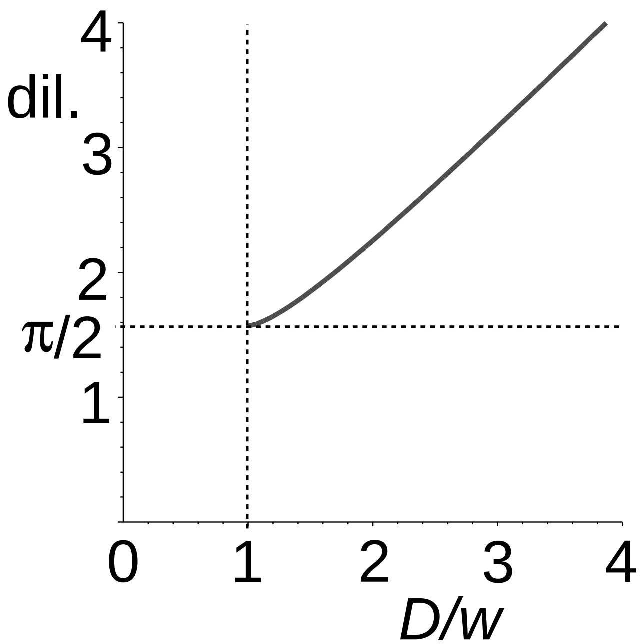

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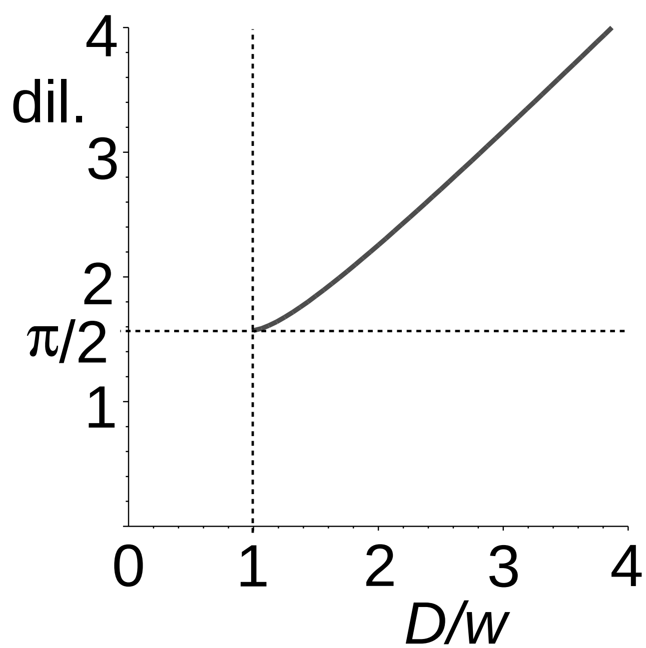

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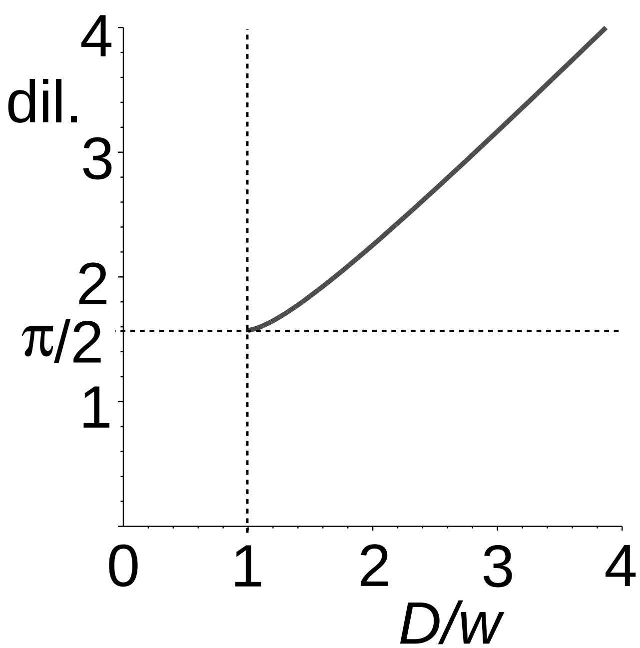

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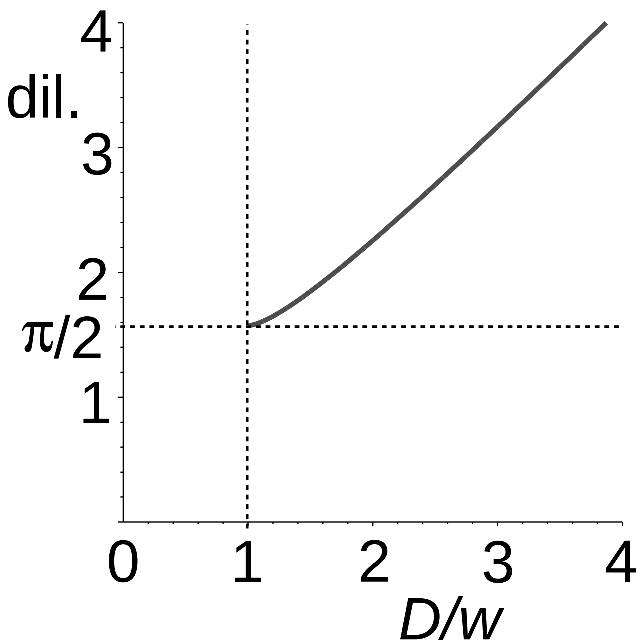

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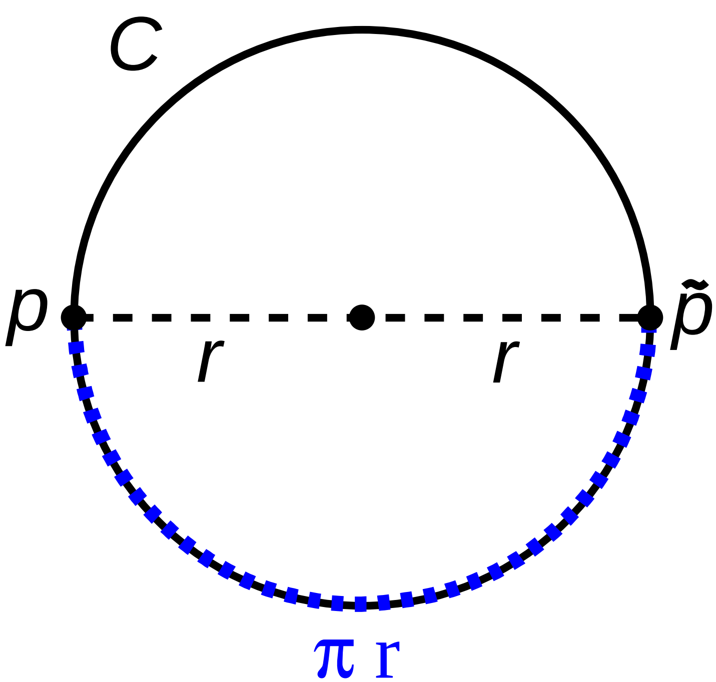

$$\delta(C) = \frac{\pi r}{2r} = \frac{\pi}{2}$$

Thank You!

This page is dedicated to Annette who forced me to add it.

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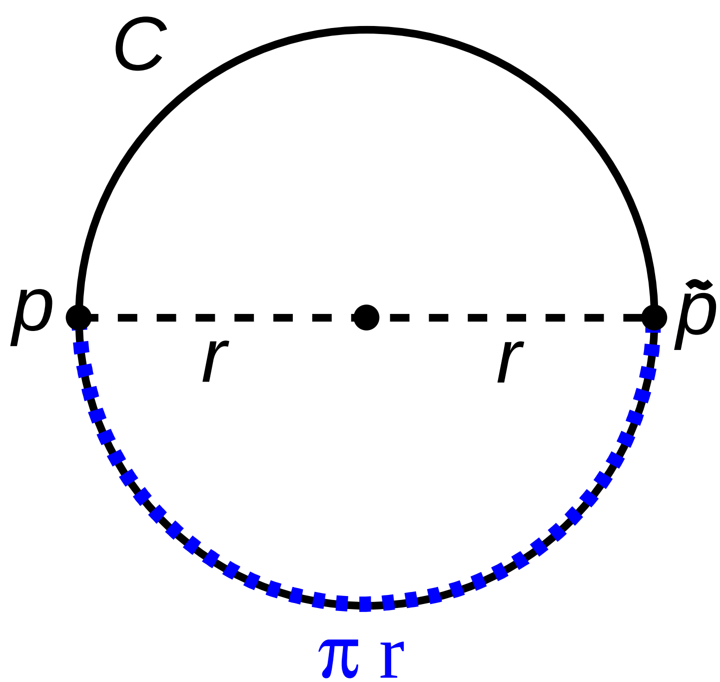

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