Discrete and Computational Geometry, SS 18
Exercise Sheet "4": WSPD Construction
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Thursday 10th of May**.
- *You may work in groups of at most two participants.*
- *You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.*

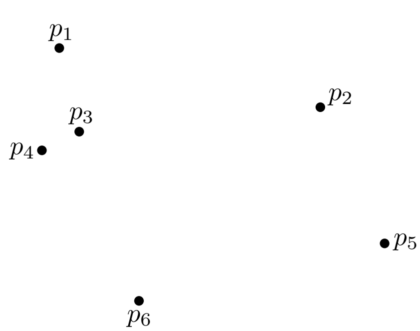
Exercise 10: WSPD example (4 Points)

Consider the point set $S \subset \mathbb{R}^2$ depicted two times below. Use the algorithm presented in the lecture to construct a WSPD of S, given the separation ratio $s = 1$.

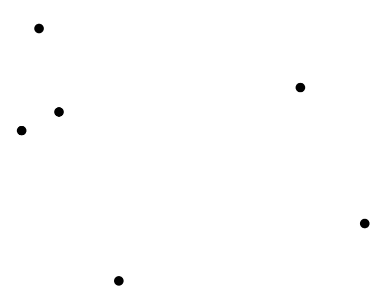
Start with computing the split-tree, and draw the resulting bounding boxes.

Use these bounding boxes to construct the WSPD. You may assume that the procedure FindPairs(v,w) only verifies if the two point sets S_v and S_w are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box and whose radius stems from half of the diameter of the boxes.

1)



2)



Exercise 11: Spanner construction by WSPD (4 Points)

Consider the point set below and construct a spanning tree T with spanning ratio $t = 5$ by using a corresponding WSPD.

Start with computing the split-tree, and draw the resulting bounding boxes. Compute the corresponding value for s

Use the bounding boxes to construct the WSPD. You may assume that the procedure FindPairs(v,w) only verifies if the two point sets S_v and S_w are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box and whose radius stems from half of the diameter of the boxes.

Exercise 12: WSPD complexity (4 Points)

Consider a WSPD $\{\{A_1, B_1\}, \ldots, \{A_m, B_m\}\}$ for a set S of n points.

a) Let $s > 4$. How many pairs (A_i, B_i) does the WSPD f at least have? I.e., present a lower bound for m! (Hint: Consider the spanner construction from the WSPD.)

b) Present a construction scheme of a point set S for any n so that the lower bound for m of part a) is tight!

c) Let $Z := \sum_{i=1}^{m}(|A_i| + |B_i|)$. Prove an upper bound for Z in O-notation depending on n.

d) Show by some examples that the upper bound from c) cannot be improved!