# Online Motion Planning MA-INF 1314

Summersemester 17
Escape Paths/Alternative Measure

Elmar Langetepe

University of Bonn

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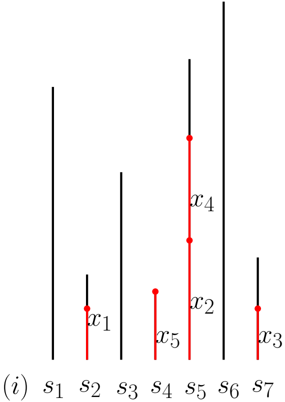
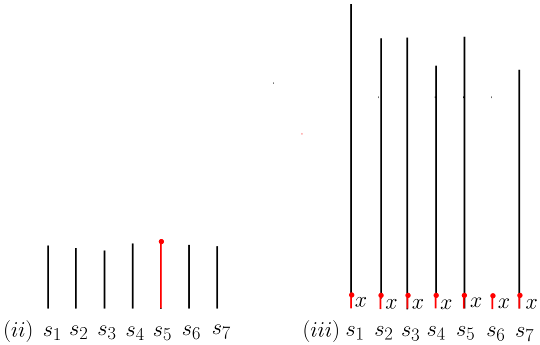
# Rep.: Different performance measures

- Set $L_m$ of $m$ line segments $s_i$ of unknown length $|s_i|$
- Dark corridors, escape, digging for oil
- Test corridors successively
- $s_{j_1}$ up to a certain distance $x_1$, then $s_{j_2}$ for another distance $x_2$ and so on



# Rep.: Partially informed

- Assume distribution is known!
- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Extreme cases! Good strategies!



# Rep.: Optimal strategy for this case

**Theorem:** For a set of sorted distances $F_m$ (i.e. $f_1 \geq f_2 \geq \cdots \geq f_m$) we have

$$\text{maxTrav}(F_m) := \min_i i \cdot f_i \,.$$

Proof:

- Arbitrary strategy $A$
- Less than $\min_i i \cdot f_i$ means less than $j \cdot f_j$ for any $j$
- Visiting depth $d_1 \geq d_2 \geq \cdots \geq d_m$
- Not reached $f_1$ by $d_1$,
  not reached $f_2$, since $d_1 + d_2 < 2f_2$ and $d_2 \leq d_1$ and so on
- Not successful!
- $\min_i i \cdot f_i$ always sufficient!

# Online Strategy

- $F_m$ with $f_1 \geq f_2 \geq \cdots \geq f_m$ not known
- Compete against $\text{maxTrav}(F_m) := \min_i i \cdot f_i$
- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \ldots$
- For any round $c$ from left to right:
  Path length of segment $i$ is *extended* up to distance $\left\lfloor \frac{c}{i} \right\rfloor$

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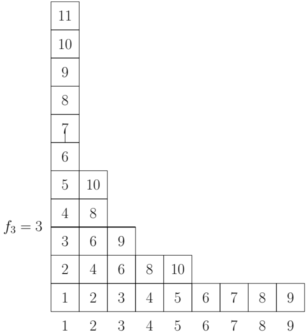


# Rep.: Online Strategy!

**Theorem:** Hyperbolic traversal algorithm solves the multi-segment escape problem for any list $F_m$ with maximum traversal cost bounded by

$$D \cdot (\text{maxTrav}(F_m) \ln(\min(m, \text{maxTrav}(F_m)))$$

for some constant $D$.

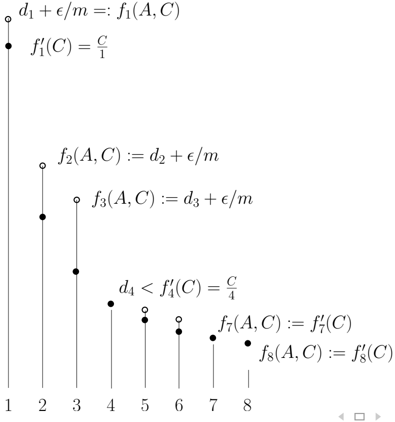
Proof:(W.l.o.g. $F_m$ integers)

- Let $\min_i i \cdot f_i = j \cdot f_j$ for some $j$
- $c$ with $c = j \cdot f_j$ exists (Round c)
- Overoll cost:

$$\sum_{t=1}^{m} \left\lfloor \frac{c}{t} \right\rfloor \leq \sum_{t=1}^{\min(m,c)} \frac{c}{t} \leq c + \int_{1}^{\min(m,c)} \frac{c}{t}\, dt = c(1 + \ln \min(m, c))\,.$$

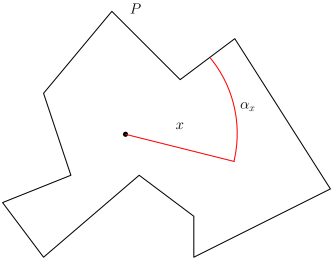
# Rep.: Matches Lower bound!

**Theorem:** For any deterministic online strategy $A$ that solves the multi-segment escape problem we can construct input sequences $F_m(A, C)$ so that $A$ has cost at least $d \cdot C \ln \min(C, m)$ and $\mathrm{maxTrav}(F_m(C, A)) \leq C$ holds for some constant $d$ and arbitrarily large values $C$.

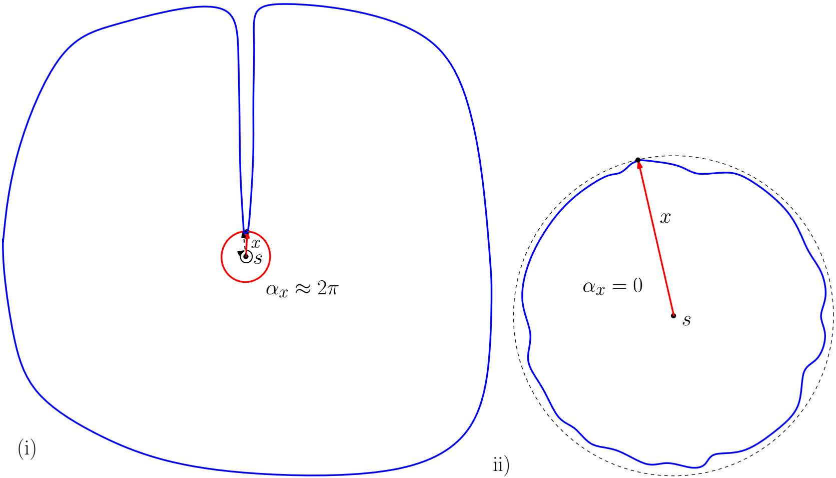


# Rep.: Different performance measure: Simple Polygon

- Simple polygon, escape path unknown
- Searching for different cost measure
- Polygonal extension of the list search problem
- Distance to the boundary $x$ (estimation, given)
- Simple circular strategy $x(1 + \alpha_x)$



- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1 + 2\pi)$, $x_2(1 + 0)$

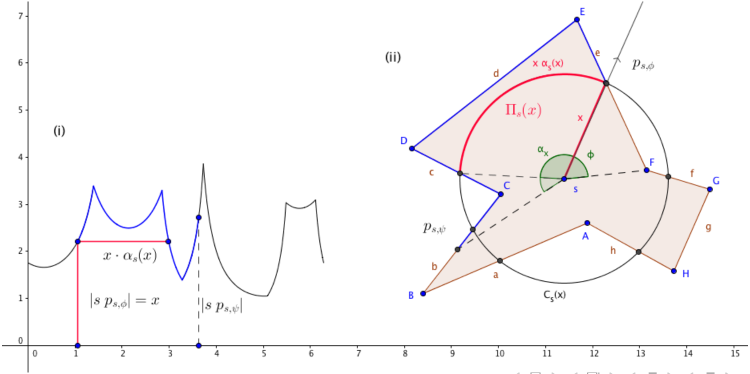


# Circular strategy: Star shaped polygon

- Optimal circular espape path for $s \in P$: $\Pi_s(x)$
- For any distance $x$ a worst-case $\alpha_s(x)$
- In total: $\min_x \; x(1+\alpha_s(x))$

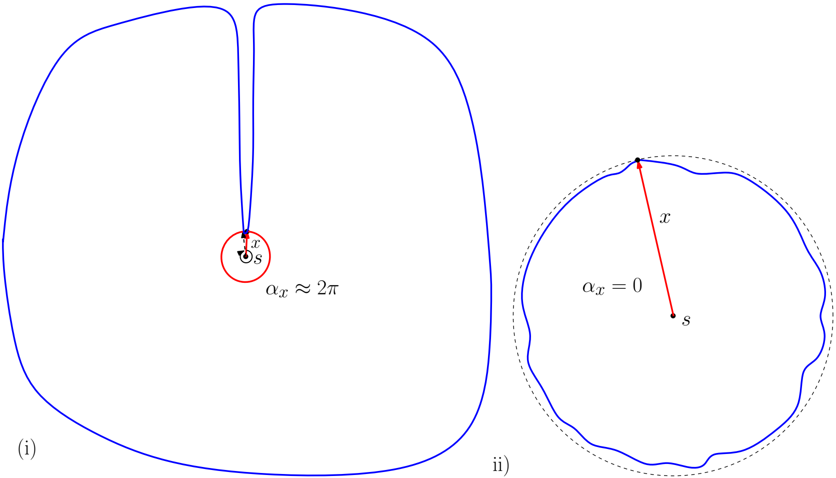
$$\Pi_s := \min_x \Pi_s(x) = \min_x x(1+\alpha_s(x)) \; .$$

- Radial dist. function interpretation: Area plus height!



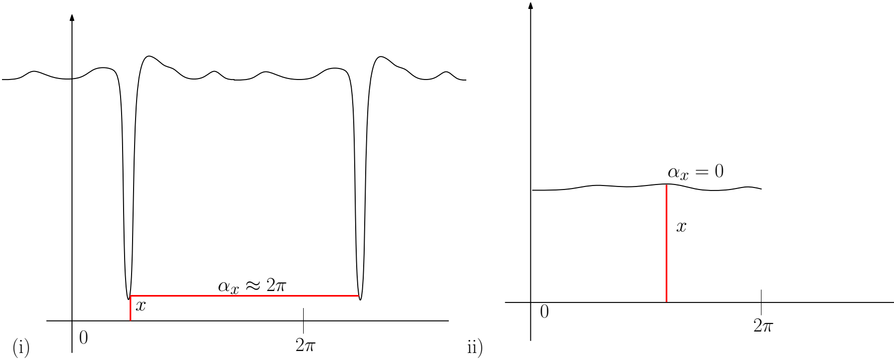
# Extreme cases: Radial dist. function

- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1+2\pi)$, $x_2(1+0)$



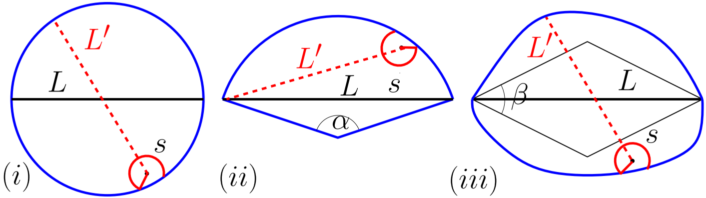
# Radial distance function of extreme cases

- Optimal circular espape path
- Hit the boundary by 90 degree wedge
- Area plus height! $\min_x \; x(1+\alpha_x)$



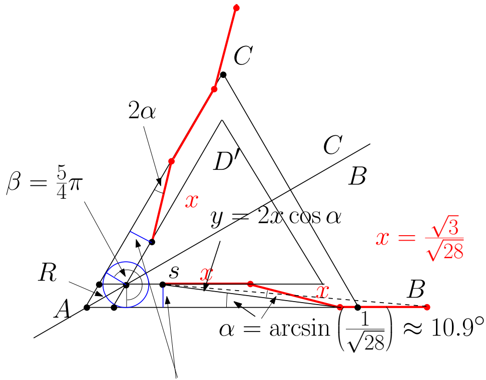
# Different justifications

- Simple, computation (polynomial), star-shaped vs. convex
- Natural extension of the discrete certificate (Kirkpatrik)
- Outperforms escape paths for known cases (diameter)



# Outperforms Zig-Zag path

- For any position, better than the Zig-Zag path
- Formal arguments!
- Zig-Zag cannot end in farthest vertex: Region *R*!

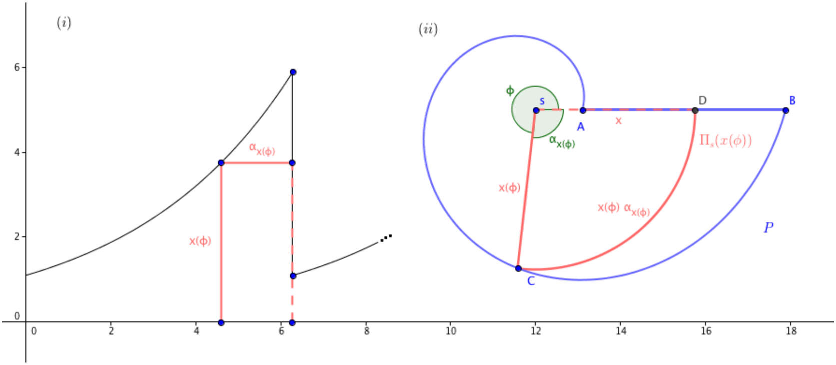



$$d = y \sin\alpha = x \sin 2\alpha \approx 0.1214$$

$$0.125 \times (5\pi/4 + 1) < 2x = 2\frac{\sqrt{3}}{\sqrt{28}}$$

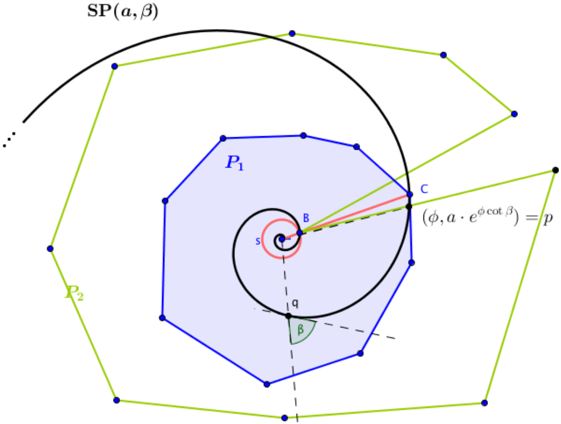
# Interesting example

- Distance distribution exactly resembles the polygon
- Analogy to discrete case! Sorting!
- Log. spiral $\alpha_x$ for any $x$ is known:
  $x(\phi) \cdot (1 + \alpha_{x(\phi)})$ with $\alpha_{x(\phi)} = 2\pi - \phi$ and $x(\phi) = A \cdot e^{\phi \cot \beta}$



# Online Approximation!

- Inside a polygon $P$ at point $s$, totally unknown
- Leave the polygon, compare to certificate path for $s \in P$
- Dovetailing strategy (discr. case)! Now spiral strategy $(a, \beta)$!



# Analysis of a spiral strategy!

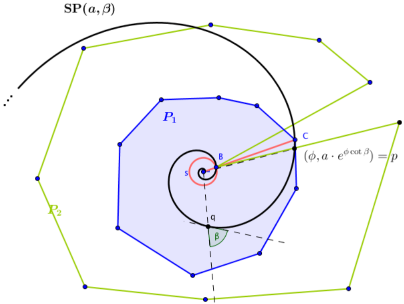
- Assume certificate: $x(1+\alpha_x)$ for $s$
- Spiral reach distance $x = a \cdot e^{(\phi-\alpha_x)\cot(\beta)}$ at angle $\phi$
- Worst-case success at angle $\phi$! (Increasing for $\alpha_x$ distances!)
- Ratio:

$$f(\gamma, a, \beta) = \frac{\frac{a}{\cos\beta} \cdot e^{\phi \cot\beta}}{a \cdot e^{(\phi-\gamma)\cot\beta}(1+\gamma)} = \frac{e^{\gamma\cot\beta}}{\cos\beta(1+\gamma)} \text{ for } \gamma \in [0, 2\pi]$$

- $\gamma$ represents possible $\alpha_x$!
- $(\beta, a)$ represents the spiral strategy!
- Independent from $a$!
- How to choose $\beta$?

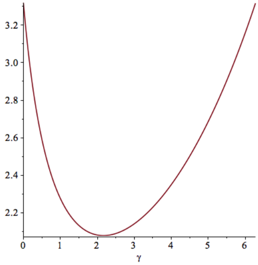
# How to choose $\beta$?

- Ratio: $f(\gamma,\beta) = \frac{e^{\gamma \cot \beta}}{\cos \beta (1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- Balance: Choose $\beta$ s.th. extreme cases have the same ratio
- $f(0,\beta) = \frac{1}{\cos \beta} = \frac{e^{2\pi \cot \beta}}{\cos \beta (1+2\pi)} = f(2\pi, \beta)$
- $\beta = \operatorname{arccot}\left(\frac{\ln(2\pi+1)}{2\pi}\right) = 1.264714\ldots$



# Balance the extreme cases!

- $\beta := \text{arccot}\left(\frac{\ln(2\pi+1)}{2\pi}\right) = 1.264714\ldots$
- Ratio: $f(\gamma,\beta) = \frac{e^{\gamma \cot \beta}}{\cos \beta (1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- $f(0,\beta) = f(2\pi,\beta) = 3.31864\ldots$
  and $f(\gamma,\beta) < 3.31864\ldots$ for $\gamma \in (0, 2\pi)$



# Spiral strategy for $\beta = 1.264714\ldots$

**Theorem:** There is a spiral strategy for any unknown starting point $s$ in any unknown environment $P$ that approximates the certificate for $s$ and $P$ within a ratio of 3.31864.