

Online Motion Planning MA-INF 1314

Searching

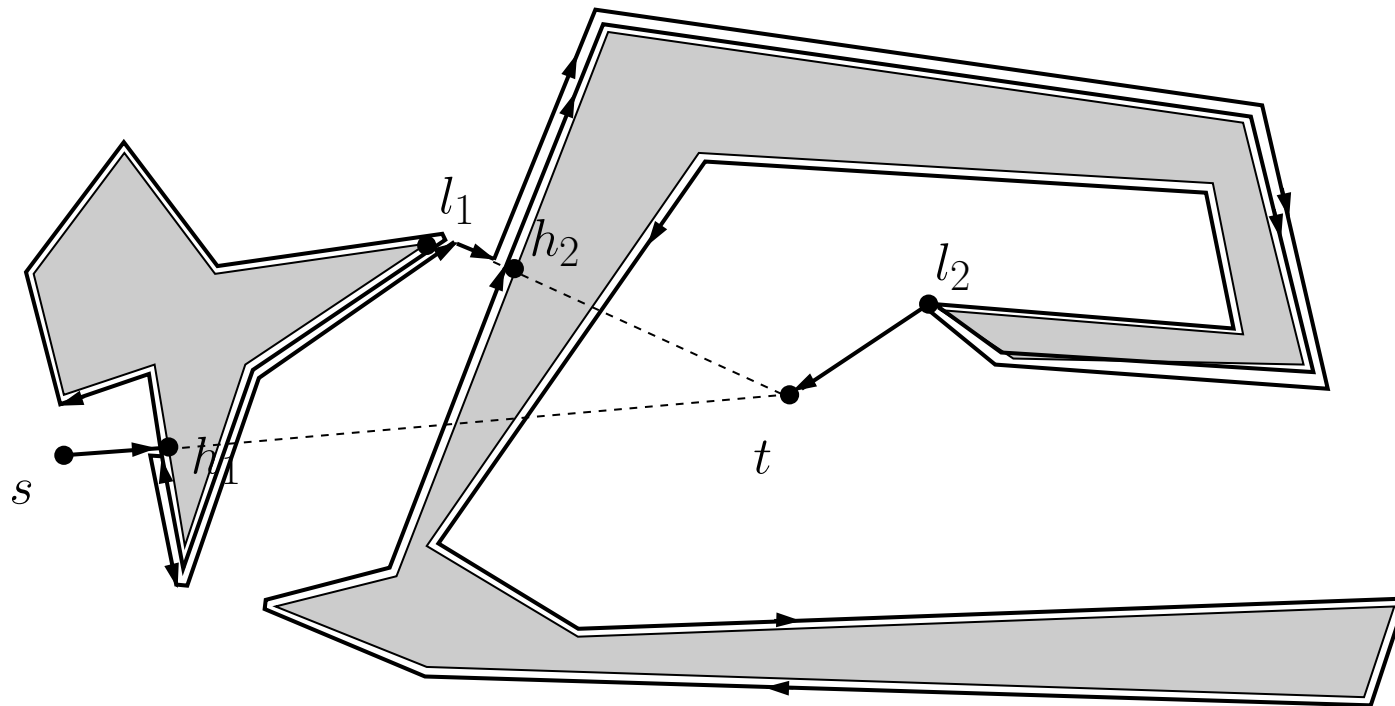
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Rep.: Navigation

- Touch sensor, Target coordinates, Start s , Target t , Storage,
- Sojourner■
- Actiony: ■
 - Move toward the target■
 - Move along the boundary ■
 - Sequence of Leave-Points l_i , Hit-Points h_i ■

Rep.: BUG1 Strategy: Lumelsky/Stepanov

Toward target, surround obstacle, best leave point, toward target!

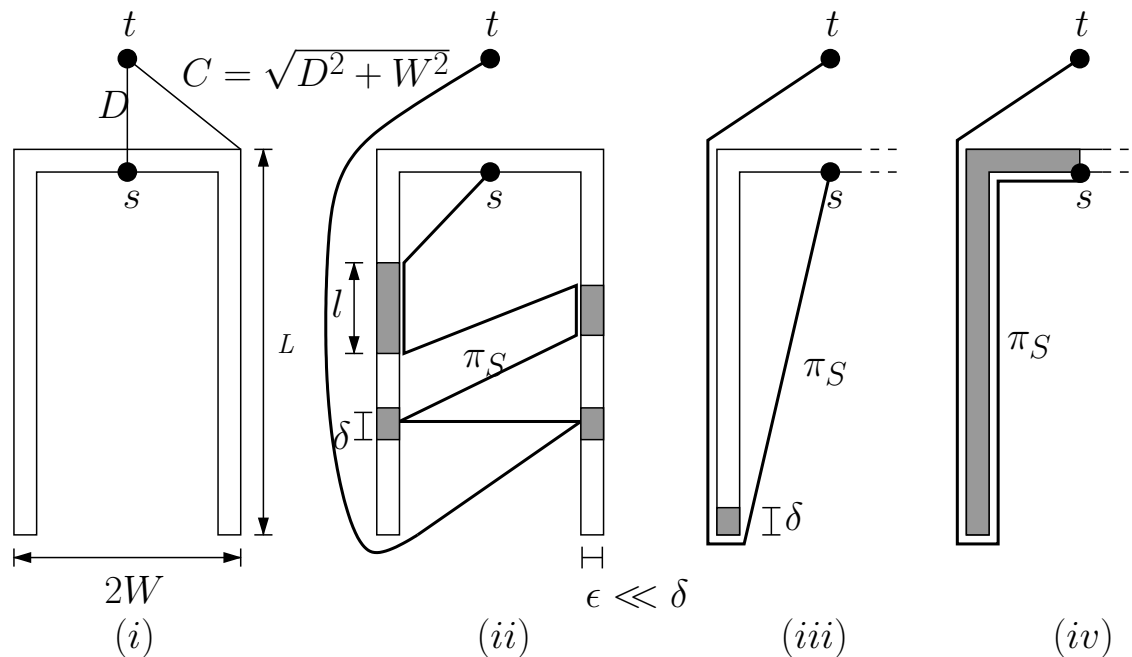


Rep: Analysis BUG1 Strategy

- **Theorem** Strategy Bug1 is correct! ■
- **Theorem** Successful Bug1-path Π_{Bug1} from start s to target t :
 $|\Pi_{\text{Bug1}}| \leq D + \frac{3}{2} \sum_i \text{UP}_i$. ■
- **Theorem** For any strategy S , for arbitrary large $K > 0$, there exists examples for any $D > 0$, such that for any arbitrarily small $\delta > 0$ we have: $|\Pi_S| \geq K \geq D + \sum \text{UP}_i - \delta$. ■
- **Korollar** Bug1 is $\frac{3}{2}$ -competitive against any *online* strategy! ■

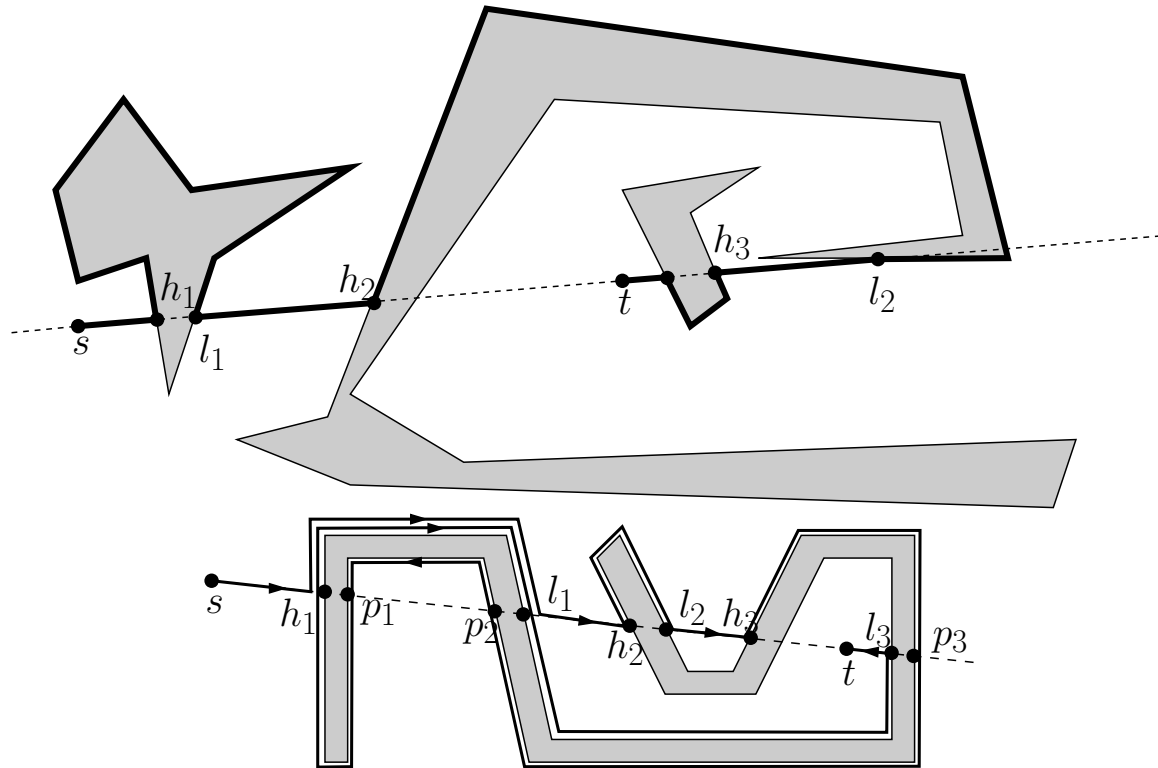
$$\text{Rep. LB: } |\Pi_S| \geq K \geq D + \sum \text{UP}_i - \delta$$

- Virtual horse-shoe , width $2W$, thickness $\epsilon \ll \delta$, length L , dist. D
- Virtual gets concrete by touch
- Roughly surround any obstacle, by any strategy!



Rep.: BUG2 Strategy

Line G passing st , toward target, surround obstacle, shorter distance on G , toward target!

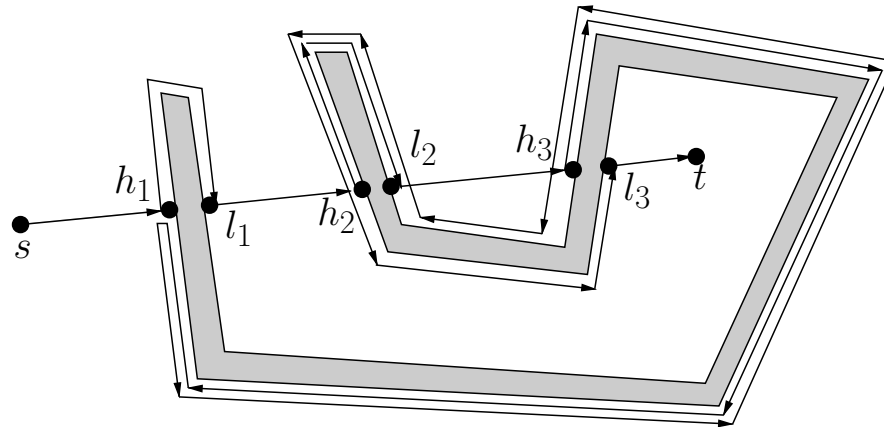


Rep.: Analysis BUG2 Strategy

- **Lemma** Let n_i denote the number of intersection of G with relevant obstacle P_i . Bug2 meets any point on P_i at most $\frac{n_i}{2}$ times.
- **Corollar** Bug2 is correct!
- **Theorem** Bug2-path Π_{Bug2} from s to t . We have:
$$|\Pi_{\text{Bug2}}| \leq D + \sum_i \frac{n_i \text{UP}_i}{2}.$$

Rep.: Change I

Change I, use former Leave/Hit Points once for !

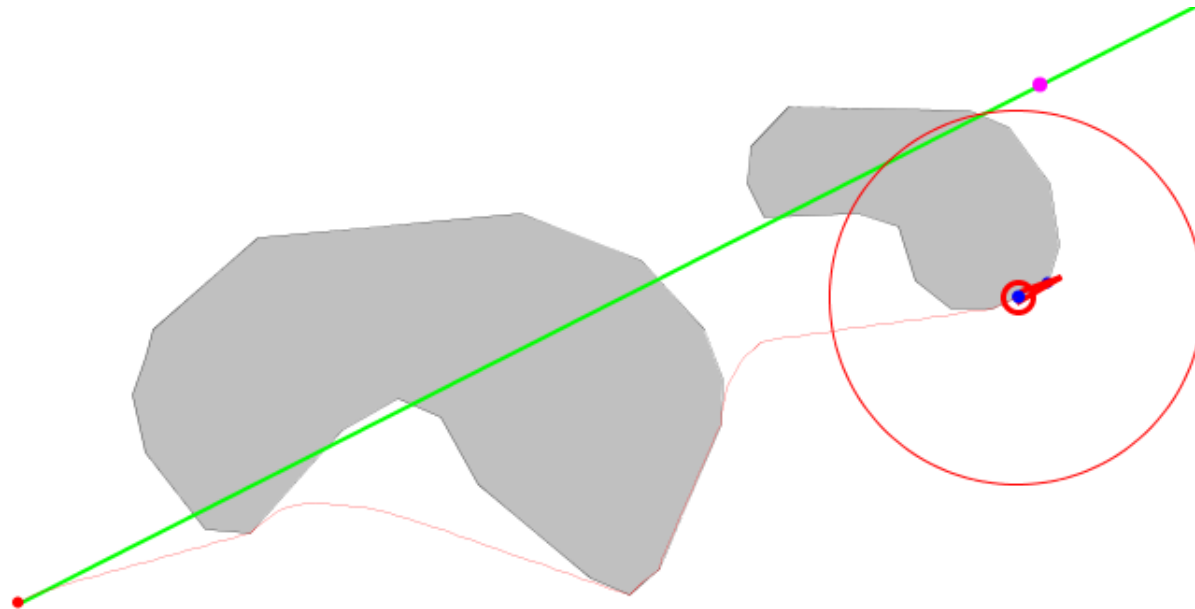


Theorem: Change I requires at most path length $|\Pi_{\text{Change I}}| \leq D + 2 \sum_i \text{UP}_i$. This is a tight bound! ■

Exercise! ■

Different models

- Sensor with range: Circle around curr. point
- ● Short-cut for BUG2: VisBug
- Many others

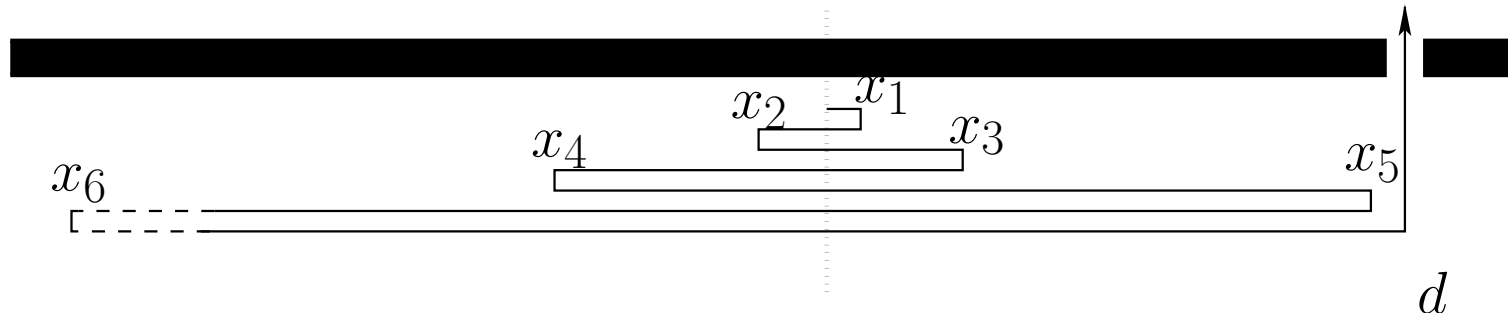


Searching for a goal!

- Coordinates of the target unknown: Searching vs. Navigation■
- Polygonal environment■
- Full sight: Visibility polygon■
- **Def.** Let P be a simple polygon and r a point with $s \in P$. The visibility polygon of r w.r.t. P , $\text{Vis}_P(r)$, is the set of all points $q \in P$, such that the segment $\overline{r\bar{q}}$ is fully inside P .■
- Alg. Geom.: Compute in $O(n)$ time! Offline!■

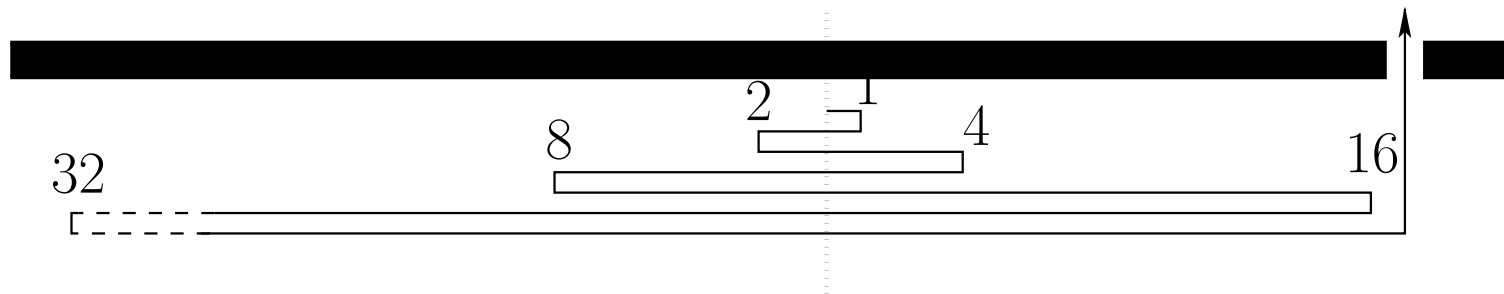
Corridors (without sight)

- 2-ray search: Find door along a wall! ■
- Compare to shortest path to the door, competitive? ■
- Reasonable strategy: Depth x_1 right, depth x_2 left and so on ■
- Start-situation: $2x_1 \geq C\epsilon$, for any $C > 0$ ex. ϵ ■
- Additive constant or goal is at least step 1 away! ■
- Local worst-case, not visited at d , once back! ■
- Find strategy, such that: $\sum_{i=1}^{k+1} 2x_i + x_k \leq Cx_k$ ■



Corridors

- Worst-case, not visited at d , once back!
- Find strategy, such that: $\sum_{i=1}^{k+1} 2x_i + x_k \leq Cx_k$ ■
- Minimize: $\frac{\sum_{i=1}^{k+1} 2x_i + x_k}{x_k} = 1 + 2\frac{\sum_{i=1}^{k+1} x_i}{x_k}$ ■
- $x_i = 2^{i-1}$, gives ratio $C = 9$ ■
- Proof: Blackboard! ■



Theorem Opt. of exponential solution: Gal 1980

- Strategy: Sequence $X = f_1, f_2, \dots$ ■
- ● Minimize functional $F_k(f_1, f_2, \dots) := \frac{\sum_{i=1}^{k+1} f_i}{f_k}$ for alle k ■
- More precisely $\inf_Y \sup_k F_k(Y) = C$ und $\sup_k F_k(X) = C$ ■
- In general: Functional F_k continuous/unimodal: Unimodal:
 $F_k(A \cdot X) = F_k(X)$ and $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$ ■
- Some other helpful conditions! ■
- I.e.: $F_{k+1}(f_1, \dots, f_{k+1}) \geq F_k(f_2, \dots, f_{k+1})$ ■
- **Theorem** Exponential function minimizes F_k :

$$\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$$

mit $A_a = a^0, a^1, a^2, \dots$ und $a > 0$. ■

Example: Exponential function

- $F_k(f_1, f_2, \dots) := \frac{\sum_{i=1}^{k+1} f_i}{f_k}$ for all k . ■
- Unimodal $F_k(A \cdot X) = F_k(X)$ and $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$? ■
- $\frac{\sum_{i=1}^{k+1} A \cdot f_i}{A \cdot f_k} = \frac{\sum_{i=1}^{k+1} f_i}{f_k}$
- $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$? ■
- Follows from $\frac{a}{b} \geq \frac{c}{d} \Leftrightarrow \frac{a+c}{d+b} \leq \frac{a}{b}$ ■
- Simple equivalence ! ■
- Optimize: $f_k(a) := \frac{\sum_{i=1}^{k+1} a^i}{a^k}$ ■
- Minimized by $a = 2$ ■

Theorem Gal 1980

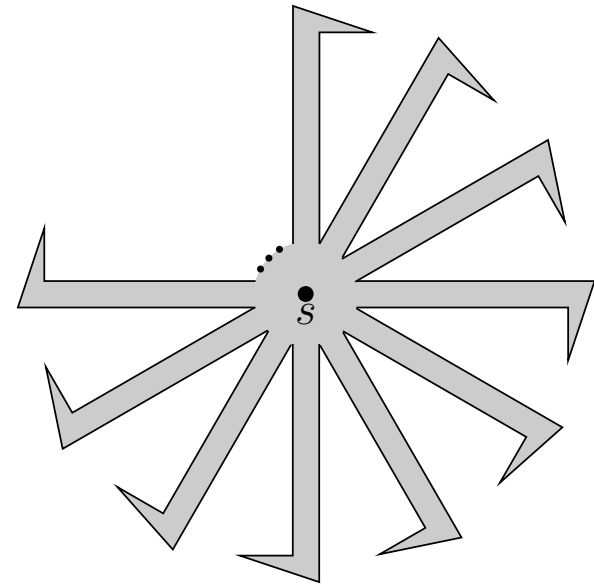
If functionally F_k has the following properties:

- i) F_k is continuous,
- ii) F_k is unimodal: $F_k(A \cdot X) = F_k(X)$ and $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$,
- iii) $\liminf_{a \mapsto \infty} F_k \left(\frac{1}{a^{k+i}}, \frac{1}{a^{k+i-1}}, \dots, \frac{1}{a}, 1 \right) = \liminf_{\epsilon_{k+i}, \epsilon_{k+i-1}, \dots, \epsilon_1 \mapsto 0} F_k \left(\epsilon_{k+i}, \epsilon_{k+i-1}, \dots, \epsilon_1, 1 \right)$,
- iv) $\liminf_{a \mapsto 0} F_k \left(1, a, a^2, \dots, a^{k+i} \right) = \liminf_{\epsilon_{k+i}, \epsilon_{k+i-1}, \dots, \epsilon_1 \mapsto 0} F_k \left(1, \epsilon_1, \epsilon_2, \dots, \epsilon_{k+i} \right)$,
- v) $F_{k+1}(f_1, \dots, f_{k+i+1}) \geq F_k(f_2, \dots, f_{k+i+1})$.

Then: $\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$ with $A_a = a^0, a^1, a^2, \dots$ and $a > 0$.

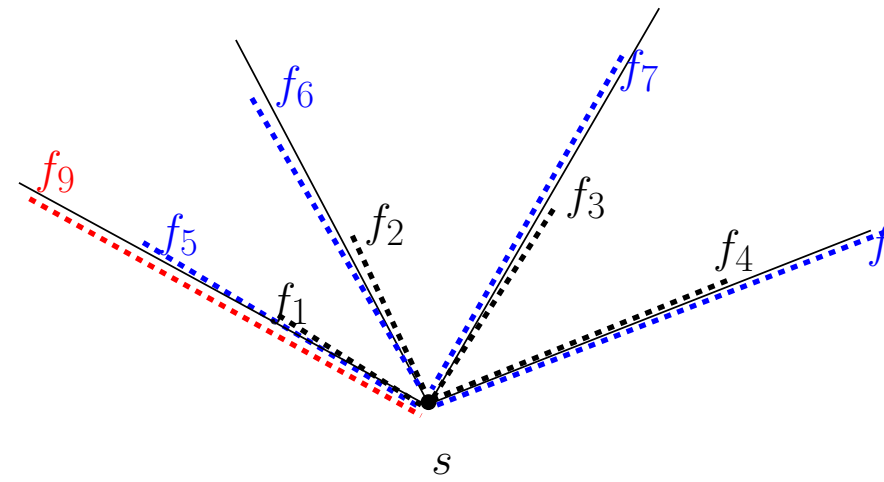
Application m-ray search

- Arbitrary m , not competitive, Fig.!
- $2m - 1$ vs. $1!$
- Fixed m , infinite rays!
- Ass.: Rays in fixed order and increasing depth
- Tupel (f_j, J_j) : depth, next visit!



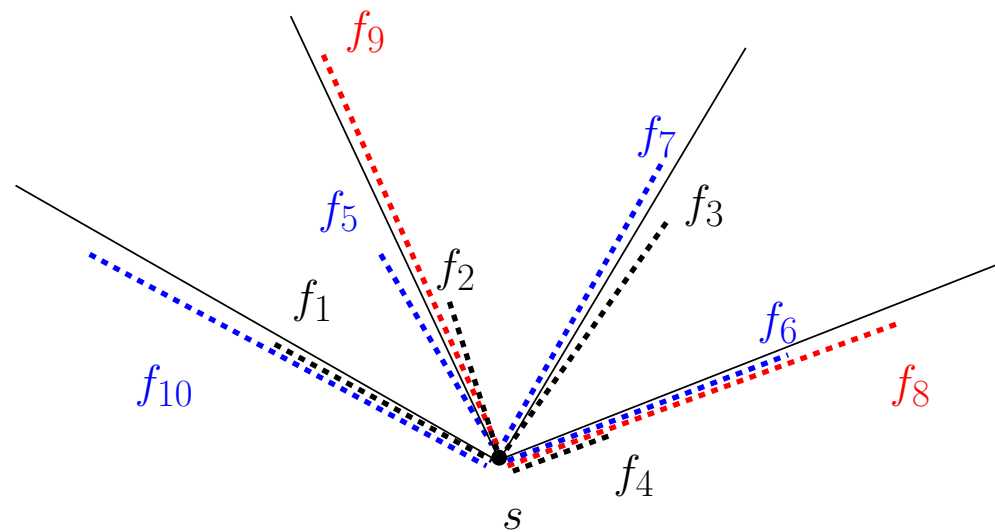
Anwendung m-Wege Suche

- Ass.: (f_j, J_j) , $J_j = j + m$, $f_j \geq f_{j-1}$ ■
- Visit rays in fixed order, increasing depth ■
- $F_k(f_1, f_2, \dots) := \frac{f_k + 2 \sum_{i=1}^{k+m-1} f_i}{f_k}$ für alle k . ■
- (Gal) Exp.-function minimizes F_k :
 $\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$
 with $A_a = a^0, a^1, a^2, \dots$ and $a > 1$,
 optimal $a = \frac{m}{m-1}$ ■
- Ratio: $C = 1 + 2m \left(\frac{m}{m-1} \right)^{m-1}$ opt. ■



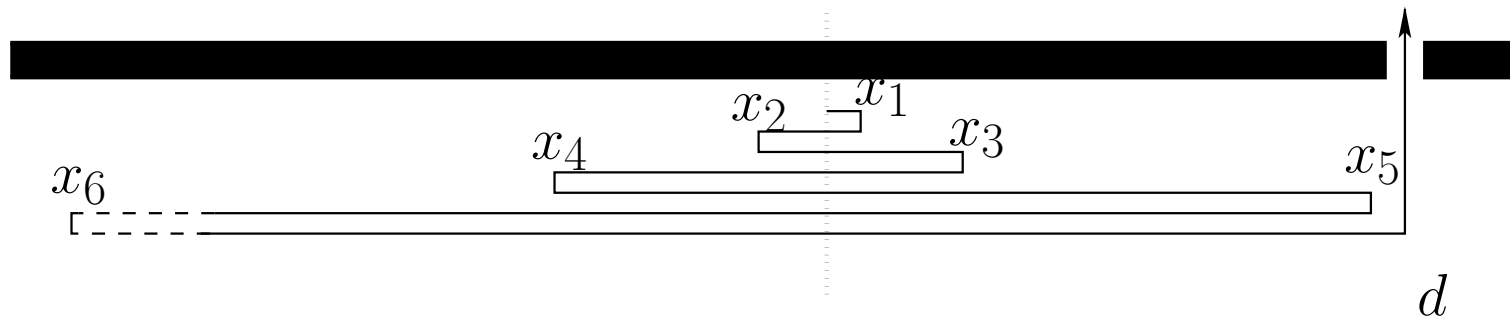
m-ray search

- **Lemma** There is an optimal m-ray search strategy (f_1, f_2, \dots) that visits the rays in a fixed order and with increasing depth. ■
- periodic and monotone: (f_j, J_j) , $J_j = j + m$, $f_j \geq f_{j-1}$ ■
- Second part: Proof blackboard! Change strategy! Conditions! ■



Other approach: Optimality for equations!

- Reasonable strategy, ratio: $\frac{\sum_{i=1}^{k+1} 2x_i + x_k}{x_k} = 1 + 2\frac{\sum_{i=1}^{k+1} x_i}{x_k}$ ■
- Ass.: C optimal, $\frac{\sum_{i=1}^{k+1} x_i}{x_k} \leq \frac{(C-1)}{2}$ ■
- There is strategy $(x'_1, x'_2, x'_3 \dots)$ s. th. $\frac{\sum_{i=1}^{k+1} x'_i}{x'_k} = \frac{(C-1)}{2}$ for all k ■
- Monotonically increasing in x'_j ($j \neq k$), decreasing in x'_k ■
- First k with: $\frac{\sum_{i=1}^{k+1} x_i}{x_k} < \frac{(C-1)}{2}$, decrease x_k ■
- $\frac{\sum_{i=1}^k x_i}{x_{k-1}} < \frac{(C-1)}{2}$!, x_{k-1} decrease etc., monotonically decreasing sequence, bounde, converges! Non-constructive! ■



Other approach: Optimality for equations!

- Set: $\frac{\sum_{i=1}^{k+1} x'_i}{x'_k} = \frac{(C-1)}{2}$ for all k ■
- $\sum_{i=1}^{k+1} x'_i - \sum_{i=1}^k x'_i = \frac{(C-1)}{2} (x'_k - x'_{k-1})$ ■
- Thus: $C' (x'_k - x'_{k-1}) = x'_{k+1}$, Recurrence!■
- Solve a recurrence! Analytically! Blackboard!■
- Characteristical polynom: No solution $C' < 4$ ■
- $x'_i = (i + 1)2^i$ with $C' = 4$ is a solution! Blackboard! ■Optimal!■

