Selected Topics in Algorithmics, SS15
Exercise Sheet "2": Triangulation and Planar Convex Hull
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Wednesday 13th of May, 14:30 pm**. *There is a letterbox in front of room E.01 in the LBH building.*
- *You may work in groups of at most two participants.*

Exercise 4: Triangulation by Conflict Lists (4 Points)

Given a set N of n points in the plane, a triangulation $T(N)$ of N is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to maintain the planarity. Let S_1, S_2, . . . , S_n be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $T(N)$ by computing $T(N^3)$, $T(N^4)$, $\ldots$,$T(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $T(N^{i+1})$ from $T(N^i)$ by adding S_{i+1}. (Hint: Add three dummy points, p_1, p_2, and p_3, in the infinity such that the outer boundary of $T(N^i \cup \{p_1, p_2, p_3\})$ is a triangle whose vertices are p_1, p_2, and p_3 for $1 \leq i \leq n$. Then when inserting S_{i+1}, $0 \leq i \leq n-1$, S_{i+1} is inside a triangle of $T(N^i \cup \{p_1, p_2, p_3\})$, and we separate the triangle into three triangles by S_{i+1}.)

1. Describe the insertion of S_{i+1}
2. Define a conflict relation between a triangle in $T(N^i)$ (i.e., $T(N^i \cup \{p_1, p_2, p_3\})$) and a point in $N \setminus N^i$
3. Prove the expected cost of inserting S_{i+1} to be $O(\frac{n}{i+1})$ (backward analysis for deleting S_{i+1} from $H(N^{i+1})$) and the expected cost of construction $T(N)$ to be $O(n \log n)$

Exercise 5: Planar Convex Hull by Conflict Lists (4 Points)

Given a set N of n half-planes in the plane, a convex hull $H(N)$ of N is the intersection of N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N. and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}.

1. Describe the insertion of S_{i+1}
2. Define a conflict relation between a vertex of $H(N^i)$ and a half-plane in $N \setminus N^i$
3. Prove the expected cost of inserting S^{i+1} to be $O(\frac{n}{i+1})$ (backward analysis for deleting S_{i+1} from $H(N^{i+1})$) and the expected cost of construction $H(N)$ to be $O(n \log n)$.

Exercise 6: Triangulation (History Graph) (4 Points)

Given a set N of n points in the plane, a triangulation $H(N)$ of N is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to main the planarity. Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the history graph. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}. (Hint: You can use the three dummy points as Exercise 4.)

1. Describe the parent and child relation in the history graph.
2. Describe the insertion of S_{i+1} using the history graph.
3. Prove the expected cost of inserting S^{i+1} to be $O(\log i)$ and the expected cost of construction $T(N)$ to be $O(n \log n)$