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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 10

Problem 1

Consider an arbitrary binary optimization problem with linear objective $c^T x$ and solution set $\mathcal{S} \subseteq \{0,1\}^n$ as discussed in Chapter 7. Recall that the winner gap Δ is defined as

$$\Delta := cx^* - cx^{**}$$

where x^* is an arbitrary optimal solution and x^{**} is a solution that is optimal amongst all solutions in $\{x \in \mathcal{S} \mid x \neq x^*\}$. Find better upper bounds on $\Pr(\Delta \leq \epsilon)$ than the bound provided by Lemma 7.3 for the following scenarios:

1. The c_i are numbers from $[1, e]$ that are chosen independently from the distribution with the density

$$f(x) = \begin{cases} \frac{1}{x} & \text{for all } x \in [1, e] \\ 0 & \text{else.} \end{cases}$$

2. The c_i are ϕ-perturbed numbers from $[0, 1]$.

Problem 2

Assume that c is ϕ-perturbed, and we additionally know that all c_i are drawn independently from the same distribution with density function $f(x)$, where

1. $f(x)$ is the density of the uniform distribution over the interval $[4, 4+u]$ for a constant $u > 0$.
2. $f(x) = \begin{cases} x^2 \cdot \frac{3}{u^3} & \text{for } x \in [0, u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (0, \infty)$.
3. $f(x) = \begin{cases} \frac{1}{x} \cdot \frac{1}{\ln u} & \text{for } x \in [1, u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (1, \infty)$.

For all three cases, do the following: Compute (a best possible) ϕ. Then give a best possible upper bound $\nu_\epsilon(u)$ for the probability to draw a number from a given fixed interval of width $\epsilon \in (0, 1)$.

Assume someone tells us that c is 3-perturbed. Based on this fixed $\phi = 3$, for which of the three scenarios do we get the largest (i.e. worst) $\nu_\epsilon(u)$?