# Multi-list Traversal Strategies (cf. [1])

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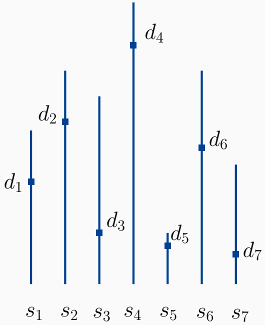
# The multi-list traversal problem (MLTP)




## Definition (traversal costs)

$\Lambda = \{\lambda_1, \lambda_2, \dots \lambda_m\}$ lists of length $\lambda_i \in \mathbb{N}$.
For ordering $(\lambda_1, \lambda_2, \dots \lambda_m)$ of $\Lambda$, the traversal costs of a strategy $\mathcal{S}$ are:

$$\mathsf{TC}(\mathcal{S}, (\lambda_1, \lambda_2, \dots \lambda_m)) = \sum_{1 \le i \le m} d_i,$$

$\mathcal{S}$ traversed $i$th list up to depth $d_i \le \lambda_i$.

## Definition (multi-list traversal problem - MLTP)

Given: Set $\Lambda$ of $m$ lists, each of unknown length.
Aim: Reach end of one list (with small traversal costs).

Note: no costs for switching lists; traversal of a list can be continued.

# Applying an alternative cost measure

1. Consider partially informed variant of MLTP

   Find reasonable strategy. (fixed depth traversal FDT)

   Define cost measure. ($\xi_\Lambda, \overline{\xi}_\Lambda$)

   Justification of the strategy/cost measure.

2. Reconsider uninformed variant of MLTP

   Suggest online strategy. (hyperbolic traversal HT)

   Prove competitiveness w.r.t. new cost measures.

# Consider partially informed variant of MLTP

# A simple and reasonable strategy: fixed depth traversal (FDT)

Given: Set $\Lambda$ of $m$ lists of known length, but unknown ordering.

Aim: Reach end of one list with small traversal costs.

$\Longrightarrow$ Lower bound for traversal costs is $\min_{1 \leq i \leq m} \lambda_i$.

$\Longrightarrow$ Any strategy that traverses every list up to depth $d \geq \min_{1 \leq i \leq m} \lambda_i$ is successful.

**Fixed-depth-traversal**

**Input:** Set $\Lambda$ of $m$ lists, fixed depth $d \in \mathbb{N}_0$

**for** $i$ from 1 to $m$ **do**
    traverse list $\lambda_i$ up to depth $d$;
**end for**

# The alternative cost measure - worst case

**Definition (intrinsic maximum traversal costs)**

The maximum traversal costs are defined as

$$\mathsf{MTC}_\Lambda(\mathsf{FDT}(d)) := \max_{\pi \in S_m} \mathsf{TC}(\mathsf{FDT}(d), \pi(\Lambda)).$$

The intrinsic maximum traversal costs are defined as

$$\xi_\Lambda := \min_{1 \le k \le m} \mathsf{MTC}_\Lambda(\mathsf{FDT}(\lambda_k)).$$

**Theorem (cf. [1], Theorem 1)**

*Reorder s.th.* $\lambda_1 \ge \lambda_2 \ge \dots \ge \lambda_m$, *then*

$$\xi_\Lambda = \min_{1 \le i \le m} i \cdot \lambda_i$$

$$i_\Lambda := \operatorname*{argmin}_{1 \le i \le m} i \cdot \lambda_i$$

$\rightsquigarrow$ Best FDT-strategy for $\Lambda$ in the worst case.

# The alternative cost measure - average case

**Definition (intrinsic average traversal costs)**

The average traversal costs are defined as

$$\mathsf{ATC}_\Lambda(\mathsf{FDT}(\lambda_k)) := \operatorname*{avg}_{\pi \in S_m} \mathsf{TC}(\mathsf{FDT}(\lambda_k), \pi(\Lambda)).$$

The intrinsic average traversal costs are defined as

$$\bar{\xi}_\Lambda := \min_{1 \le k \le m} \mathsf{ATC}_\Lambda(\mathsf{FDT}(\lambda_k)).$$

**Theorem (cf. [1], Lemma 1)**

*Reorder s.th.* $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_m$, *then*

$$\bar{\xi}_\Lambda \le \min_{1 \le i \le m} \frac{(m+1) \cdot \lambda_i}{m - i + 2}$$

$$\bar{i}_\Lambda := \operatorname*{argmin}_{1 \le i \le m} \frac{\lambda_i}{m-i+2}$$

$\rightsquigarrow$ Good FDT-strategy for $\Lambda$ in the average case.

# Justification of FDT

1. The competitive ratio of breadth-first traversal ($=$ FDT$(\lambda_m)$) is $\Omega(m)$ and the competitive ratio of depth-first traversal ($=$ FDT$(\lambda_1)$) is unbounded. [1]
2. No traversal strategy that is successful on all permutations of $\Lambda$, has fewer traversal costs than $\xi_\Lambda$ in the worst case. [2]
3. Any traversal strategy that terminates with traversal costs of at most $\bar{\xi}_\Lambda/3$ on all presentations of $\Lambda$, fails with probability $1/2$ on a random presentation of $\Lambda$. [3]
   $\rightsquigarrow \bar{\xi}_\Lambda$ is $\theta\left(\frac{(m+1)\cdot\lambda_{\bar{i}_\Lambda}}{m-\bar{i}_\Lambda+2}\right)$

[1] cf. [1], Theorem 3
[2] cf. [1], Proof of Theorem 1
[3] cf. [1], Lemma 2 and Theorem 2

# Reconsider uninformed variant of MLTP

# Hyperbolic traversal (HT)

Given: Set $\Lambda$ of $m$ lists of unknown length.

Aim: Reach end of one list with small traversal costs.

**Hyperbolic-traversal**

**Input:** List $\Lambda$

```
c ← 1;
while no list fully explored do
    for i from 1 to m do
        explore list i up to depth ⌊c/i⌋;
    end for
    c ← c + 1;
end while
```

# Competitiveness of HT

**Theorem**

*HT solves MLTP with* $O\left(\xi_\Lambda \cdot \ln(\min(m, \xi_\Lambda))\right)$ *maximum traversal costs.* [4]

**Theorem**

*HT solves the MLTP with* $O\left(\overline{\xi}_\Lambda \cdot \ln(\min(m, \overline{\xi}_\Lambda))\right)$ *in the average traversal costs.* [5]

## Optimality

As D. Kirkpatrick shows in [1], HT is also optimal w.r.t. the alternative cost measure.

[4] cf. [1], Theorem 4
[5] cf. [1], Theorem 6

# References I

D. G. Kirkpatrick.
**Hyperbolic dovetailing.**
In *Algorithms - ESA 2009, 17th Annual European Symposium, Copenhagen, Denmark, September 7-9, 2009. Proceedings*, pages 516--527, 2009.