Institut für Informatik
Prof. Dr. Heiko Röglin
Clemens Rösner

Probabilistic Analysis
of Algorithms
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Problem Set 10

For an undirected graph $G = (V, E)$ with weights $w : E \to \mathbb{R}_{\geq 0}$ the *Maximum-Cut Problem* is the problem of finding a partition of V into disjoint sets V_0 and V_1 that maximizes

$$w(V_0, V_1) := \sum_{\substack{e=\{u,v\}\in E \\ u\in V_0 \wedge v\in V_1}} w(e)$$

over all possible partitions. We call a partition (V_0, V_1) also a *cut* and we say that $w(V_0, V_1)$ is the weight of the cut (V_0, V_1).

We consider the simple local search algorithm FLIP for the Maximum-Cut Problem that starts with an arbitrary cut (V_0, V_1) and iteratively increases the weight of the cut by moving one vertex from V_0 to V_1 or vice versa, as long as such an improvement is possible. For $i \in \{0, 1\}$ and a vertex $v \in V_i$ the switch corresponding to v is moving v from V_i to V_{1-i}, which creates a new cut $(V_i', V_{1-i}') = (V_i \setminus \{v\}, V_{1-i} \cup \{v\})$. A switch is improving if it increases the weight of the cut, i.e. $w(V_i', V_{1-i}') > w(V_0, V_1)$. The algorithm FLIP stops when the current cut does not admit an improving switch anymore.

Problem 1

Show that FLIP outputs a cut whose weight is at least half the weight of the maximum cut.

Problem 2

Show a pseudo-polynomial upper bound on the running time of FLIP for instances in which all weights are integers.

Problem 3

(a) Assume that the weights $w : E \to [0, 1]$ are ϕ-perturbed numbers. Give an upper bound on the expected number of iterations of FLIP on instances in which G has maximal degree δ.

(b) For which values of δ is the expected number of iterations from part (a) polynomial.

Problem 4

Let V be a set of n points in $[0, 1] \times [0, 1]$. For every pair of points $u, v \in V$, let $d(u, v)$ be defined as the Euclidean distance between u and v. Show that the length of the optimal TSP tour with respect to d is $O(\sqrt{n})$.