

Discrete and Computational Geometry, WS1415  
Exercise Sheet “6”: Dilations  
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until **Tuesday 25th of November 14:00 pm**. There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, [hilko.delonge@uni-bonn.de](mailto:hilko.delonge@uni-bonn.de), if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at <https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom>*

**Exercise 16:      Visibility and maximum dilation              (4 Points)**

The graph-theoretic dilation of a planar graph  $G$  with vertex set  $V$  is

$$\delta_{graph}(G) := \sup_{p \neq q \in V} \frac{|\pi_p^q|}{|pq|}$$

where  $|pq|$  is the euclidean distance from  $p$  to  $q$  and  $|\pi_p^q|$  is the length of a shortest path in  $G$  from  $p$  to  $q$ .

- Construct a planar graph  $G$  where the maximum graph-theoretic dilation of  $G$  is attained by a pair of non-visible vertices.
- Recall the definition of geometric dilation of a planar graph. Prove that for a planar, simply connected graph  $G$  there is always a pair of points  $p, q \in G$  with maximal dilation so that  $p$  and  $q$  are co-visible.

**Exercise 17: Dilation and AVDs (4 Points)**

The decision problem for the geometric dilation of a polygonal chain  $C = (p_1, p_2, \dots, p_n)$  was translated into the problem of tracing the chain  $C$  through an additively weighted Voronoi diagram.

We proved the following statement: *If for a point  $(q_x, q_y) \in C$  appearing after  $C_i = (p_1, p_2, \dots, p_i)$  on  $C$ , the point  $(q_x, q_y, a_q)$  with  $a_q := \frac{|C_{p_1}^q|}{K}$  lies below any cone  $K_{p_i}$  starting at height  $a_{p_i} := \frac{|C_{p_1}^{p_i}|}{K}$  at  $p_i$ , the dilation  $\delta_C(p_i, q)$  between  $q$  and  $p_i$  is smaller than  $K$ .*

- Why do we trace the chain  $p_i, p_{i+1}, \dots, p_n$  through the additively weighted Voronoi diagram of  $p_1, p_2, \dots, p_i$  with weights  $a_{p_i}$ ?
- Why can we compute the Voronoi diagram for all points  $p_1, p_2, \dots, p_n$  with weights  $a_{p_i}$  and trace the complete chain (for one direction) only once? Or the other way round: Why is it not necessary to incrementally compute the Voronoi diagrams for  $p_1, p_2, \dots, p_i$  and successively trace the chains  $p_i, p_{i+1}, \dots, p_n$ ?