Exercise 16: Randomized Incremental Algorithm for Abstract Voronoi Diagrams (History Graph) (4 Points)

Consider an admissible bisecting curve system \((S, \mathcal{J})\), and make a general position assumption that no four curves in \(\mathcal{J}\) intersect at the same point. Let \(s_1, s_2, \ldots, s_n\) be a random sequence of \(S\), and let \(R^i = \{\infty, s_1, s_2, \ldots, s_i\}\). Please develop a randomized algorithm to construct the abstract Voronoi diagram \(V(S)\) by computing \(V(R^2), V(R^3), \ldots, V(R^n)\) iteratively using the history graph. In other words, for \(i \geq 2\), obtain \(V(R^{i+1})\) from \(V(R^i)\) by insertion \(s_{i+1}\). Let a configuration be a Voronoi edge of \(V(R^i)\), for \(2 \leq i \leq n\).

1. Define the parent and child relation between a configuration in \(V(R^i) \setminus V(R^{i+1})\) and a configuration in \(V(R^{i+1}) \setminus V(R^i)\).

2. Please prove that if a site conflicts a configuration, there exists a path from the root of the history graph to the configuration along which all configuration is in conflict with the site.

3. Prove that the expected time complexity of inserting \(s^i\) is \(O(\log i)\).
Exercise 17: Removal of General Position Assumption (4 Points)

Consider an admissible bisecting curve system \((S, J)\) without the general position assumption that no four curves in \(J\) intersect at the same point. In other words, more than three curves in \(J\) can intersect at the same point, and the degree of a Voronoi vertex can be more than three. Please complete the following

- Use a constant number of sites to define a Voronoi edge, i.e., formulate a configuration for a Voronoi edge. Note that a site can appear more than once in a configuration.
- Please describe how to update the conflict graph after inserting \(s\) into \(V(R)\).

Exercise 18: Karlsruhe metric (4 Points)

The Karlsruhe metric, also known as the Moscow metric, is a distance measure in a radial city where there is a city center, and roads either circumvent the center or are extended from the center. The distance \(d_K(p_1, p_2)\) between two points is \(\min(r_1, r_2) \times \delta(p_1, p_2) + |r_1 - r_2|\) if \(0 \leq \delta(p_1, p_2) \leq 2\) and \(r_1 + r_2\), otherwise, where \((r_1, \psi_1)\) are the polar coordinates of \(p_1\) with respect to the center, and \(\delta(p_1, p_2) = \min(|\psi_1 - \psi_2|, 2\pi - |\psi_1 - \psi_2|)\) is the angular distance between the two points. Please prove the bisecting curve system in the Karlsruhe metric to be admissible.