Discrete and Computational Geometry Winter term 2016/2017
Exercise Sheet 08
University Bonn, Institute of Computer Science I

Deadline: Tuesday 13.12.2016, until 12:00 Uhr Discussion: 9.01. - 13.01.

- *Please give your solutions directly to the tutor or put them in the postbox at LBH next to E.01 until the deadline. Write your names well visible and readable on the first page. If your solutions consists of multiple pages, make sure they are well connected.*
- *It is possible to submit in groups of up to three people.*

Aufgabe 1: Sweep Algorithm for stabbing intervals (4 Points)

Consider the problem of finding a minimal stabber (transversal) in dimension 1:

Given a finite set $\mathcal{R}$ of n intervals on the x-axis and a set $\mathcal{P}$ of m points on the x-axis, find a minimum subset $\mathcal{P}_{min} \subseteq \mathcal{P}$ such that each interval $\mathcal{I} \in \mathcal{R}$ contains at least one point of $\mathcal{P}_m in$ (i.e. $\forall \mathcal{I} \in \mathcal{R} : \mathcal{I} \cap \mathcal{P}_{min} \neq \emptyset$).

It was mentioned in the lecture that this can be solved efficiently with a sweep algorithm by adding a point every time the end of a not yet stabbed interval is reached.

Work out the details of this algorithm:

a) The content of the Sweep Status Structure (SSS)

b) The types of events in the Event Structure (ES)

c) The handling of an event (i.e. how does the SSS, ES and solution change)

d) Give the worst case running time and space requirements of your algorithm.

Aufgabe 2: VC-Dimension of stars in Graphs (4 Points)

Based on a undirected, simple graph $G = (V, E)$ we consider the set system $(X, \mathcal{F})$, where $X = E$ and $\mathcal{F}$ denotes the set of all stars of G. A *star* s_v of G is the set of all edges of G incident to v.

Which VC-Dimension does the set system $(X, \mathcal{F})$ have at most? Prove your claim.

Tip: Try using a proof by contradiction by assuming there is an $A \subseteq X$ *of size* $VCdim(\mathcal{F}+1)$ *that is shattered by* $\mathcal{F}$.

Bonus question (1 Point): Which classical graph problem do we obtain, when we try to determine the minimum transversal of $(\tilde{X}, \tilde{\mathcal{F}})$, $\tilde{X} := \mathcal{F}$, $\tilde{\mathcal{F}} := \{\{F \in \tilde{X} | x \in F\} | x \in X\}$?

Aufgabe 3: Transversals and packings (4 Points)

Consider the following set of sets $\mathcal{F}$:

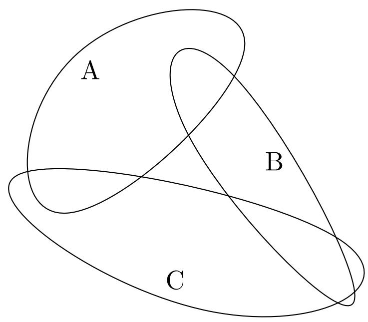

Abbildung 1: Three intersecting sets

a) Give a minimum transversal $\mathcal{T}$ and the resulting τ as well as a maximum packing $\mathcal{M}$ and the resulting ν. Argue why those are minimum/maximum examples.

a) Give a minimum fractional transversal φ and the resulting τ^* as well as a maximum fractional packing ψ and the resulting ν^*. Argue why those are minimum/maximum examples.