Discrete and Computational Geometry Winter term 2016/2017
Exercise Sheet 06
University Bonn, Institute of Computer Science I

Deadline: Tuesday 29.11.2016, until 12:00 Uhr Discussion: 5.12. - 9.12.

- *Please give your solutions directly to the tutor or put them in the postbox at LBH next to E.01 until the deadline. Write your names well visible and readable on the first page. If your solutions consists of multiple pages, make sure they are well connected.*
- *It is possible to submit in groups of up to three people.*

**Aufgabe 1: Minkowski Sum (4 Points)**

Let $P$ and $Q$ be convex polygons with $n$ and $m$ edges respectively.
Prove: The Minkowski-Sum $P \oplus Q$ is a convex polygon with at most $n + m$ edges!

**Aufgabe 2: Morphing Lemma (4 Points)**

During the proof of the Brunn-Minkowski-Inequality we used the following equality for morphing from a convex $A \subseteq \mathrm{R}^d$ into a convex $B \subseteq \mathrm{R}^d$ as a lemma:

$$\operatorname{conv}(\{0\} \times A, \{1\} \times B) = \bigcup_{t \in [0,1]} (\{t\} \times ((1-t)A + tB))$$

Prove the lemma!

**Aufgabe 3: Non-concave volume functions (4 Points)**

Let $A \subset \mathbb{R}^d$ be a set containing a single point and $B \subset \mathbb{R}^d$ the unit hypercube.

**a)** Give an explicit formula for the volume function $v(t) = \operatorname{vol}((1-t)A + tB)$.

**b)** Prove $v(t)^\beta$ is not concave on $[0, 1]$ for any $\beta > \frac{1}{d}$.