Aufgabe 1: Trapezoidal Decomposition Procedure (4 Points)

Apply the procedure to create a trapezoidal decomposition described in the lecture. Given the following point set \( S \) in \( \mathbb{R}^2 \):

\[
S = \{(-4, -4), (0, -3), (0, 3), (3, -6), (6, 1)\}
\]

a) Create (per Hand) the euclidean Voronoi Diagram \( V(S) \) of \( S \).

b) Create (per Hand) the trapezoidal decomposition \( T(V(S)) \), such that you can locate a query point \( p \) from \([-10, 10] \times [-10, 10]\) in the voronoi regions.

c) Create a DAG \( D(V(S)) \) for \( T(V(S)) \) as a data structure for point localisation using the incremental procedure from the lecture. Add the line segments of the Voronoi Diagram from the left to the right (ordered by the x-coordinate of their left endpoint, breaking ties by x-coordinate of the right endpoint).

d) Mark the query path for the following 3 queries in \( D(V(S)) \):
   \( p_1 = (1, 1), p_2 = (9, -8), p_3 = (-1, 0) \).
   Which voronoi regions do you get as a result?

Aufgabe 2: Trapezoidal Decomposition Properties (4 Points)

Prove the following propositions:

a) Every face \( f \) of a trapezoidal decomposition \( T(S) \) of a set \( S \) of \( n \) line segments in general position\(^1\) is bordered by one or two vertical edges and exactly two non-vertical edges.
   Tip: First prove that every \( f \) is convex.

b) The trapezoidal decomposition \( T(S) \) of a set \( S \) of \( n \) line segments in general position\(^1\) consists of at most \( 6n + 4 \) vertices and at most \( 3n + 1 \) trapezoids.

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\(^1\)The line segments are in general position, if they intersect only at endpoints and no two different endpoints (from the same or different line segments) have the same x-coordinate.
Aufgabe 3: Log* (4 Points)

Consider \( \log^* n := \min \left\{ m \in \mathbb{N}_0 \mid \log_2^{(m)}(n) \leq 1 \right\} \)

where the \( m \)-fold application of \( \log_2 \) on \( n \) is denoted as \( \log_2^{(m)}(n) \), i.e.

\[
\log_2^{(m)}(n) := \begin{cases} 
n & \text{falls } m = 0 \\
\log_2(\log_2^{(m-1)}(n)) & \text{sonst}
\end{cases}
\]

a) Prove that \( \log^* n \) is the smallest number \( m \in \mathbb{N}_0 \), such that the tower from \( m \)-many twos \( \text{tower}(m) := 2(2^{\ldots}) \) has value at least \( n \).

b) What is the smallest number \( n \in \mathbb{N} \) with \( \log^* n = 5 \)?

c) Prove \( \log^* n \in O(\log n)! \)

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\(^1\)In the lecture we defined \( \log^* n \) as \( \max \left\{ h \in \mathbb{N}_0 \mid \log_2^{(h)}(n) \geq 1 \right\} \). However both definitions differ only by \( \pm 1 \), which doesn’t matter in applications. This exercises reads much nicer this way.