Different performance measure: Simple Polygon

- Simple polygon, escape path unknown
- Searching for different cost measure
- Polygonal extension of the list search problem
- Distance to the boundary $x$ (estimation, given)
- Simple circular strategy $x(1 + \alpha_x)$
Extreme cases: Circular strategy

- Circular escape path: Distribution of the length is known
- Extreme situations: \( x_1(1 + 2\pi) \), \( x_2(1 + 0) \)
Discrete Version! Extreme Cases!

- Assume distribution is known!
- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Extreme cases! $x_1(m), x_2(1)$
Circular strategy: Star shaped polygon

- Optimal circular escape path for \( s \in P \): \( \Pi_s(x) \)
- For any distance \( x \) a worst-case \( \alpha_s(x) \)
- In total: \( \min_x x(1 + \alpha_s(x)) \)

\[
\Pi_s := \min_x \Pi_s(x) = \min_x x(1 + \alpha_s(x)).
\]

- Radial dist. function interpretation: Area plus height!

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Extreme cases: Radial dist. function

- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1 + 2\pi), x_2(1 + 0)$
Radial distance function of extreme cases

- Optimal circular escape path
- Hit the boundary by 90 degree wedge
- Area plus height! $\min_x x(1 + \alpha_x)$
Different justifications

- Simple, computation (polynomial), star-shaped vs. convex
- Natural extension of the discrete certificate (Kirkpatrik)
- Outperforms escape paths for known cases (diameter)
Outperforms Zig-Zag path

- For any position, better than the Zig-Zag path
- Formal arguments!
- Zig-Zag cannot end in farthest vertex: Region $R$!

\[ d = y \sin \alpha = x \sin 2\alpha \approx 0.1214 \]

\[ 0.125 \times (5\pi/4 + 1) < 2x = 2\frac{\sqrt{3}}{\sqrt{28}} \]
Interesting example

- Distance distribution exactly resembles the polygon
- Analogy to discrete case! Sorting!
- Log. spiral $\alpha_x$ for any $x$ is known:
  
  \[ x(\phi) \cdot (1 + \alpha_x(\phi)) \text{ with } \alpha_x(\phi) = 2\pi - \phi \text{ and } x(\phi) = A \cdot e^{\phi \cot \beta} \]
Online Approximation!

- Inside a polygon $P$ at point $s$, totally unknown
- Leave the polygon, compare to certificate path for $s \in P$
- Dovetailing strategy (discr. case)! Now spiral strategy $(a, \beta)$!
Analysis of a spiral strategy!

- Assume certificate: $x(1 + \alpha_x)$ for $s$
- Spiral reach distance $x = a \cdot e^{(\phi - \alpha_x) \cot(\beta)}$ at angle $\phi$
- Worst-case success at angle $\phi$! (Increasing for $\alpha_x$ distances!)
- Ratio:

$$f(\gamma, a, \beta) = \frac{a}{\cos \beta} \cdot e^{\phi \cot \beta} \frac{a \cdot e^{(\phi - \gamma) \cot \beta}}{a \cdot e^{(\phi - \gamma) \cot \beta}(1 + \gamma)} = \frac{e^{\gamma \cot \beta}}{\cos \beta(1 + \gamma)} \text{ for } \gamma \in [0, 2\pi]$$

- $\gamma$ represents possible $\alpha_x$!
- $(\beta, a)$ represents the spiral strategy!
- Independent from $a$!
- How to choose $\beta$?
How to choose $\beta$?

- Ratio: $f(\gamma, \beta) = \frac{e^{\gamma \cot \beta}}{\cos \beta(1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- Balance: Choose $\beta$ s.th. extreme cases have the same ratio
  
  $f(0, \beta) = \frac{1}{\cos \beta} = \frac{e^{2\pi \cot \beta}}{\cos \beta(1+2\pi)} = f(2\pi, \beta)$

- $\beta = \arccot \left( \frac{\ln(2\pi+1)}{2\pi} \right) = 1.264714\ldots$
Balance the extreme cases!

- $\beta := \arccot \left( \frac{\ln(2\pi+1)}{2\pi} \right) = 1.264714 \ldots$
- Ratio: $f(\gamma, \beta) = \frac{e^{\gamma \cot \beta}}{\cos \beta (1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- $f(0, \beta) = f(2\pi, \beta) = 3.31864 \ldots$
  and $f(\gamma, \beta) < 3.31864 \ldots$ for $\gamma \in (0, 2\pi)$
Spiral strategy for $\beta = 1.264714 \ldots$

**Theorem 76:** There is a spiral strategy for any unknown starting point $s$ in any unknown environment $P$ that approximates the certificate for $s$ and $P$ within a ratio of $3.31864$. 