Discrete and Computational Geometry, WS1516
Exercise Sheet "3": Graph and Geometric Detour
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Wednesday 18th of November, 12:00 pm**.
- *There is a letterbox in front of Room E.01 in the LBH builiding.*
- *You may work in groups of at most two participants.*

Exercise 7: Visibility and maximum Detour (4 Points)

The graph-theoretic detour of a planar graph G with vertex set V is

$$\delta_{graph}(G) := \sup_{p \neq q \in V} \frac{|\pi_p^q|}{|pq|}$$

where $|pq|$ is the euclidean distance from p to q and $|\pi_p^q|$ is the length of a shortest path in G from p to q.

- Construct a planar graph G where the maximum graph-theoretic detour of G is attained by a pair of non-visible vertices.
- Recall the definition of geometric detour of a planar graph. Prove that for a planar, simply connected graph G there is always a pair of points $p, q \in G$ with maximal detour so that p and q are co-visible.

Exercise 8: Detour and AVDs (4 Points)

The decision problem for the geometric detour of a polygonal chain $C = (p_1, p_2, \ldots, p_n)$ was translated into the problem of tracing the chain C through an additively weighted Voronoi diagram.

We proved the following statement: *If for a point* $(q_x, q_y) \in C$ *appearing after* $C_i = (p_1, p_2, \ldots, p_i)$ *on* C*, the point* (q_x, q_y, a_q) *with* $a_q := \frac{|C_{p_1}^{q}|}{K}$ *lies below any cone* K_{p_i} *starting at height* $a_{p_i} := \frac{|C_{p_1}^{p_i}|}{K}$ *at* p_i*, the detour* $\delta_C(p_i, q)$ *between* q *and* p_i *is smaller than* K*.*

- Why do we trace the chain $p_i, p_{i+1}, \ldots, p_n$ through the additively weighted Voronoi diagram of $p_1, p_2, \ldots, p_i$ with weigths a_{p_i}?
- Why can we compute the Voronoi diagram for all points $p_1, p_2, \ldots, p_n$ with weights a_{p_i} and trace the complete chain (for one direction) only once? Or the other way round: Why is it not necessary to incrementally compute the Voronoi diagrams for $p_1, p_2, \ldots, p_i$ and successively trace the chains $p_i, p_{i+1}, \ldots, p_n$?

Bonus 1: Non-Crossing Vertex-Edge Cuts (4 Points)

In the lecture, we have shown that a detour can be attained by a vertex-edge cut and if two vertex-edge cuts intersects with each other, one of them can be neglected. Please prove that the maximum number of non-crossing vertex-edge cuts is $O(n)$ for a polygonal chain of n vertices.