Exercise 4: Master Theorem I (4 Points)
Consider a function $T(\cdot)$ satisfying the following recurrence:

$$T(n) = (\ln r + 1)T(\lceil \alpha n \rceil) + O(D(n)),$$

where $r, \alpha < 1$, and $\epsilon > 0$ are constants and $D(n)$ is a function such that $D(n)/n^\epsilon$ is monotone increasing in $n$. Please prove that if $(\ln r + 1)\alpha^\epsilon < 1$, $T(n) \leq C \cdot D(n)$, where $C$ is a constant depending on $r, \alpha$, and $\epsilon$.

Exercise 5: Master Theorem II (4 Points)
Consider a function $T(\cdot)$ satisfying the following recurrence:

$$T(n) = 2T(\lceil n/2 \rceil) + O(D(n)),$$

where $D(n)/n$ is monotone increasing in $n$ and $\epsilon$ is a positive constant. Please prove the following.

- $T(n) = O(D(n) \log n)$.
- If $D(n)/n^{1+\epsilon}$ is monotone increasing in $n$ where $\epsilon > 0$, $T(n) = O(D(n))$. 

- Written solutions have to be prepared until Wednesday 11th of November, 12:00 pm.
- There is a letterbox in front of Room E.01 in the LBH building.
- You may work in groups of at most two participants.
Exercise 6: Voronoi Diagrams (4 Points)

Given a set $S$ of $n$ points in the Euclidean plane, the Voronoi diagram $V(S)$ partitions the plane into Voronoi regions $VR(p, S)$, $p \in S$, such that all points in $VR(p, S)$ share the same nearest site $p$ among $S$. We make a general position assumption that no more than three points of $S$ are located on the same circle. Let $e$, $v$, and $u$ be the numbers of edges, vertices, unbounded faces of $V(S)$.

1. Please prove $e = 3(n - 1) - u$ and $v = 2(n - 1) - u$. (Hint: use Euler’s formula)

2. Please explain that the number of vertices will not increase without the general position assumption. In other words, $v \leq 2(n - 1) - u$. 
