Discrete and Computational Geometry, WS1516
Exercise Sheet “1”: Geometry Duality and Finding the Minimum
University of Bonn, Department of Computer Science I

• Written solutions have to be prepared until Wednesday 4th of November, 12:00 pm.
• There is a letterbox in front of Room E.01 in the LBH building.
• You may work in groups of at most two participants.

Exercise 1: Geometry Duality II (4 Points)
We define a geometry duality $\Psi(\cdot)$ as follows ($O$ denotes the origin)

• For a point $p = (a, b) \in \mathbb{R}^2 \setminus O$, $\Psi(p)$ maps to the line $ax + by = 1$.
• For a line $L : ax + by = 1$, $\Psi(L)$ maps to the point $(a, b)$.

Please prove the following.

For a point $p \in \mathbb{R}^2 \setminus O$ and a line $L$ that does not pass through $O$, $p$ and $O$ are located in the same side of $L$ if and only if $\Psi(L)$ and $O$ are located in the same side of $\Psi(p)$.

Exercise 2: Geometry Duality II (4 Points)
We define a geometry duality $\Phi(\cdot)$ as follows

• For a point $p = (a, b) \in \mathbb{R}^2$, $\Phi(p)$ maps to the line $y = ax - b$.
• For a line $L : y = ax - b$, $\Phi(L)$ maps to the point $(a, b)$.
Please prove the following.

For a point \( p \in \mathbb{R}^2 \) and a nonvertical line \( L \), \( p \) lies above \( L \) if and only if \( \Phi(L) \) lies above \( \Phi(p) \).

Exercise 3: Finding the Minimum (4 Points)

Given \( r \) distinct numbers, let \((a_1, a_2, \ldots, a_r)\) be a random permutation of the \( r \) numbers. For \( i > 1 \), Let \( A_i \) be the event that \( a_i \) is smaller than all numbers in \( \{a_1, \ldots, a_{i-1}\} \). Please answer the following two questions.

- What is the probability \( \text{Prob}(A_i) \) of event \( A_i \)?
- What is the value of \( \sum_{i=2}^{r} \text{Prob}(A_i) \)?