Exercise 4: Upper bound for Shannons Mouse (4 points)

Given a grid graph $G$ over $n + 1 \geq 2$ cells we denote by $c(s)$ the cell in $G$ which is explored last by the MOUSE algorithm, given that the mouse starts at cell $s$.

Prove that for any cell $s$ in $G$, the graph $G'$ which we obtain by removing cell $c(s)$ from $G$, is a connected grid graph over $n$ cells. Then, use a Proof by contradiction to show that after at most $n \cdot 4^n$ moves, the MOUSE algorithm has successfully explored graph $G$.

Exercise 5: Shortest paths and number of edges (4 points)

Prove that the length $d(s, t)$ of any shortest path between two cells $s$ and $t$ in the first layer of a grid polygon $P$ is at most $\frac{1}{2}E(P) - 2$ (where $E(P)$ denotes the number of boundary edges of $P$).

Exercise 6: A property of simple grid polygons (4 points)

Prove that for any grid polygon $P$ that contains no narrow passages, and that contains no split cells in its first layer, the equality

$$E(P) \leq \frac{2}{3}C(P) + 6$$

is fulfilled. Here, $E(P)$ denotes the number of boundary edges and $C(P)$ the number of cells of $P$. 