Exercise 1: Basics of Competitive Analysis (4 points)

In this exercise a robot is placed in some grid cell inside an unknown grid graph $G$. The robot performs an exploration strategy $ALG$ until it has found some special item, which is located at a target destination $i$ which is also unknown to the robot. Prove the following statements assuming that $k \in \mathbb{N}$ is a constant.

a) For any (correct) exploration algorithm $ALG$, if $G$ is (unknown to the algorithm) a grid graph of at most $k$ cells, then for any constant $c > 0$, it holds that there exists a constant $\alpha$ where

\[ ALG \leq c \cdot OPT + \alpha \]

b) Prove that if the size of $G$ is not necessarily bounded, then for any constant $1 > c > 0$ there is no algorithm $ALG$ for which exists a constant $\alpha$ such that

\[ ALG \leq c \cdot OPT + \alpha \]

holds.

Please turn the page!
Exercise 2: The MOUSE Algorithm (4 points)

We consider the MOUSE Algorithm for finding a destination grid cell inside an unknown grid graph $G$. Suppose graph $G$ is a horizontal corridor consisting of $n$ cells, as shown in Figure 1. Initially, in each cell, the mark used by the algorithm points in direction NORTH.

Assuming that the destination grid cell is cell 1, answer the following questions.

a) How many moves are made by the MOUSE Algorithm before it finds the destination cell, depending on the starting cell $i$, $1 < i \leq n$? Starting from which cell maximizes this value?

b) Let $MOUSE(i)$ denote the number of steps required to find the target, starting from cell $i$, as determined in part a). Which starting position maximizes the value

$$\frac{MOUSE(i)}{OPT(i)}$$

where $OPT(i)$ denotes the number of steps required by an optimal offline strategy $OPT$, starting in cell $i$. 

Figure 1: A vertical corridor of length $n$. 

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1 2 \ldots n
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| 1 | 2 | \ldots | n |
Exercise 3:  **MOUSE using DFS**  

In order to improve the performance of the *MOUSE* Algorithm we want our mouse to explore the given grid graph using the *DFS* Algorithm. To this end, we enhance the capabilities of the mouse as follows.

First, instead of one mark each cell now has four different marks, each of which the mouse can distinguish (by its smell). Second, the mouse can now read the marks of adjacent cells, that is, of the adjacent cells in *NORTH*, *EAST*, *SOUTH* and *WEST* direction of its current cell.

Describe how the mouse can find an arbitrary destination cell by performing the *DFS* Algrorithm. It is not required to provide *(PSEUDO-)*Code. Instead, focus on how the different marks can be used to represent the information used by the *DFS* Algorithm – and how they are updated during the execution of the algorithm.

*Hint: Be careful not to use extra memory the mouse does not posess – e.g. for a recursion stack. The mouse knows its algorithm and makes decisions based on (the algorithm and on) the marks on the floor only.*