Exercise Sheet 9

Exercise 9.1: Driver behaviour functions (4 Punkte)

Analysis of topological filtrations have real-world applications. Consider the following two curves of driving speed over time for drivers on a highway - one of an experienced, relaxed driver and the other of a nervous driver.

Consider the filtration of the sublevel-sets and draw the according merge trees according to the elder rule. How can this be used to classify nervous or relaxed drivers?

Exercise 9.2: Mini Ball Algorithm (4 Punkte)

During the analysis of the running time for the MiniBall-Algorithm, we used a double recursive formula to analyse the runtime:

\[ t_j(0) = 0 \quad \text{for all } j \]
\[ t_j(n) \leq t_j(n - 1) + 1 + \frac{j}{n} t_{j-1}(n - 1) \]

Prove that \( t_j(n) \leq (j + 1)!n \).

Exercise 9.3: Randomized Backwards Analysis (4 Punkte)

Consider the standard algorithm for finding the maximum in an array with \( n \) integers, which consists of checking every entry in the array in order and keeping a \( \text{MaxSoFar} \) that holds the maximum found so far.

Let’s say checking itself is an cheap operation, but updating \( \text{MaxSoFar} \) is expensive. What is the expected number of times \( \text{MaxSoFar} \) has to be updated, if the order of the items in the array is randomized?