Exercise Sheet 6

Exercise 6.1: Vertex and chain maps (4 Punkte)

A simplicial map $f$ between simplicial complexes $K$ and $L$ maps each vertex of $K$ to a vertex of $L$ in such a way that the vertices of a simplex in $K$ are mapped to the vertices of a simplex in $L$. Prove that

$$g(\langle v_1 v_2 \ldots v_d \rangle) := \langle f(v_1) f(v_2) \ldots f(v_d) \rangle$$

if all $f(v_i)$ are different and := 0, otherwise,

defines a chain map from the chain complex of $K$ to the chain complex of $L$, and thus a homomorphism of the homology spaces.

*Hint: Ex.3 on sheet 1 might be useful*

Exercise 6.2: Contraction vs collapsing (4 Punkte)

Which 1-dimensional complexes can be

- contracted into a point?
- collapsed into a point?

Exercise 6.3: Bonus: House with two rooms (4 Punkte)

An example for a contractible, but not collapsible 2-dimensional complex is given below by the "House with two rooms". Why is it not collapsible? Can you describe a deformation retraction to a point?

*Hint: It might be easier to imagine the retraction the other way around.*