Exercise Sheet 5

Exercise 5.1: Snake Lemma representant independence (4 Punkte)
Show that the choice of $\gamma$ in the proof of the Snake-Lemma as given in the lecture is independent of the representatives $\alpha$ and $\beta$.

Exercise 5.2: Theorem of Mayer-Vietoris and Homologies (4 Punkte)
The Theorem of Mayer-Vietoris can be used to compute Homology of a topological space in a divide-and-conquer approach.
Recall: Let $K = K' \cup K''$ and $A = K' \cap K''$. By Mayer-Vietoris, there exists a long exact sequence
\[
\cdots \to H_p(K') \oplus H_p(K'') \to H_p(K) \to H_{p-1}(A) \to H_{p-1}(K') \oplus H_{p-1}(K'')
\]
Proof that the following holds for the sequence above:
\[
H_p(K) \cong \exists [H_p(K') \oplus H_p(K'')] \oplus \text{Ker} [H_{p-1}(A) \to H_{p-1}(K') \oplus H_{p-1}(K'')]
\]

Exercise 5.3: Theorem of Mayer-Vietoris application (4 Punkte)
Apply Mayer-Vietoris to determine the Betti-numbers of the Double-torus by splitting it between both holes, which corresponds to cutting along the green line in the graphic below. As a first step, consider the what $K$, $K'$, $K''$ and $A$ are in this scenario.