Exercise 22: Stabbing in 1D (4 Points)

Consider the problem of finding a minimal stabber (transversal) in dimension 1:

Given a finite set $\mathcal{R}$ of $n$ intervals on the x-axis and a set $\mathcal{P}$ of $m$ points on the x-axis, find a minimum subset $\mathcal{P}_{\min} \subseteq \mathcal{P}$ such that each interval $I \in \mathcal{R}$ contains at least one point of $\mathcal{P}_{\min}$ (i.e. $\forall I \in \mathcal{R} : I \cap \mathcal{P}_{\min} \neq \emptyset$).

It was mentioned in the lecture that this can be solved efficiently with a sweep algorithm by adding a point every time the end of a not yet stabbed interval is reached.

Work out the details of this algorithm:

a) The content of the Sweep Status Structure (SSS)

b) The types of events in the Event Structure (ES)

c) The handling of an event (i.e. how does the SSS, ES and solution change)

d) Give the worst case running time and space requirements of your algorithm.
Exercise 23: Shatter Function Lemma (4 Points)

1. Show the correctness of
\[ \binom{m-1}{i} + \binom{m-1}{i-1} = \binom{m}{i}. \]

2. Show that the bound (ii) in the Shatter Function Lemma is tight! Construct a set system $\mathcal{F}$ for all $d$ and $m$ such that $VCdim(\mathcal{F}) = d$ and $\pi_\mathcal{F}(m) = \Phi_d(m)$, where $\Phi_d(m) = \binom{m}{0} + \binom{m}{1} + \ldots + \binom{m}{d}$ holds.

3. Carify the proof detail on page 111 of the manuscript:
\[ \left(1 - \frac{d}{m}\right)^{d-m} \]
is increasing in $m$!
Figure 1: The points $p$ and $q_1$ are $L_1$-visible whereas $p$ and $q_2$ are not $L_1$-visible because the $L_1$-visibility is blocked by the horizontal $L_1$-cut of the locally $Y$-minimal vertex $v_2$.

Exercise 24: VC Dimension $L_1$-visibility (4 Points)

Consider the following notion of $L_1$-visibility inside a simple polygon $P$: Two points $p$ and $q$ inside $P$ are $L_1$-visible to each other in $P$, iff there is an $L_1$-path inside $P$ from $p$ to $q$ that is monotone in $X$- and $Y$-direction, see the Figure for some examples.

Try to find an example in order two show that the VC-Dimension of points in simple polygons is 3 (or even 4) w.r.t. $L_1$-visibility polygons of $P$!