Exercise 13: MST from WSPD? (4 Points)

Prove or disprove the following statement:
For a WSPD of a point set $S$ for $s > 4$ in dimension $d$ consider any pair $\{A_i, B_i\}$ and build the shortest edge $a_i b_i$ with $a_i \in A_i$, $b_i \in B_i$. The given collection of edges will result in the Minimum-Spanning-Tree of the point set $S$, i.e., the edge-length minimal tree that connects all points in $S$.

Exercise 14: Packing argumentation (4 Points)

Consider two reals $l$ and $L$ such that $0 < l \leq L \leq 1$.

How many disjoint axis-parallel boxes $[a_1, b_1] \times [a_2, b_2] \times \ldots \times [a_d, b_d]$, with $\min_i |b_i - a_i| \geq l$ and $\max_i |b_i - a_i| \leq L$ can intersect with the hypercube $[0, 1]^d$?

Give a proof for a non-trivial (reasonably small) upper bound depending on $l$, $L$ und $d$.

Exercise 15: Nearest neighbors by WSPD (4 Points)

Construct the WSPD for $s = 3$ and the set $S = \{0, 4, 5, 7, 12, 13, 14, 16\}$ in $\mathbb{R}^1$ by making use of a single split tree.

Compute the 3 nearest neighbors for any point by the strategy presented in the lecture.