Exercise 10: WSPD example (4 Points)

Consider the point set \( S \subset \mathbb{R}^2 \) depicted two times below. Use the algorithm presented in the lecture to construct a WSPD of \( S \), given the separation ratio \( s = 1 \).

Start with computing the split-tree, and draw the resulting bounding boxes. Use these bounding boxes to construct the WSPD. You may assume that the procedure \( \text{FindPairs}(v,w) \) only verifies if the two point sets \( S_v \) and \( S_w \) are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box and whose radius stems from half of the diameter of the boxes.
Exercise 11: Spanner construction by WSPD (4 Points)

Consider the point set below and construct a spanning tree $T$ with spanning ratio $t = 5$ by using a corresponding WSPD.

Start with computing the split-tree, and draw the resulting bounding boxes. Compute the corresponding value for $s$

Use the bounding boxes to construct the WSPD. You may assume that the procedure $\text{FindPairs}(v, w)$ only verifies if the two point sets $S_v$ and $S_w$ are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box and whose radius stems from half of the diameter of the boxes.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{exercise11.png}
\end{figure}
Exercise 12: WSPD complexity (4 Points)

Consider a WSPD \( \{\{A_1, B_1\}, \ldots, \{A_m, B_m\}\} \) for a set \( S \) of \( n \) points.

a) Let \( s > 4 \). How many pairs \((A_i, B_i)\) does the WSPD at least have? I.e., present a lower bound for \( m \) depending on \( n \). ( Hint: Consider the spanner construction from the WSPD.)

b) Present a construction scheme of a point set \( S \) for any \( n \) so that the lower bound for \( m \) of part a) is tight!

c) Let \( Z := \sum_{i=1}^{m}(|A_i| + |B_i|) \). Prove an upper bound for \( Z \) in \( O \)-notation depending on \( n \).

d) Show by some examples that the upper bound from c) cannot be improved!