Online Motion Planning MA-INF 1314
Smart DFS

Elmar Langetepe
University of Bonn
Repetition!

- SmartDFS: DFS returnpath and components
- Simple design, complicated analysis: $C + \frac{1}{2}E - 3$
- Structural property: l-Offset, l-Layer
- Edgelemma: l-Offset 8l edges less
- Pathlemma: Shortest path $\leq \frac{1}{2}E(P) - 2$
- Induction: Decompose at splitcell!
- Excesslemma: $\text{excess}(P) \leq \text{excess}(P_1) + \text{excess}(K_2 \cup \{c\}) + 1$
- Induction over number of splitcells
Definition $P_1$, $P_2$ and ExcessLemma!

- Splitcell $c \in K_2 \cup \{c\}$ first!
- $P_2$ $q$-Offset of $K_2 \cup \{c\}$ $q = l = 1$, then $P_1 := ((P \setminus P_2) \cup Q) \cap P$!
- $\text{excess}(P) \leq \text{excess}(P_1) + \text{excess}(K_2 \cup \{c\}) + 1$
Repetition: Edges of $P$ und $Q$

**Lemma:** $P, P_1, P_2$ und $Q$ as given. For the number of edges we have $E(P_1) + E(P_2) = E(P) + E(Q)$. 

![Diagram showing the edges and layers of $P$ and $Q$.]
Repetition: Exploration Theorem

**Theorem:** SmartDFS explores a simple gridpolygon $P$ with $C$ cells and $E$ boundary edges with at most $C + \frac{1}{2}E - 3$ steps.

Proof: Induction over number of components

- **Induction base:** One component
- Visit cells: $C - 1$, back to start
- Shortest path Lem: $\frac{1}{2}E(P) - 2 + C - 1 = C + \frac{1}{2}E - 3$
- And so on by induction!
Repetition! Wavefront and competitive ratio

- Wavefront Algorithmu (Lee): $O(n)$, $n$ cells
- Comp. Factor: $S(P) \leq \frac{4}{3} C(P) - 2$ (Lower bound $\frac{7}{6}$)
- Observation: Optimally in narrow passages!
Repetition

- Analyse polygons $P_i$, $i = 1, \ldots, k$
- Induction over split-cells
- Induction-Base: No split-cell in layer 1.
- **Lemma** $E(P) \leq \frac{2}{3}C(P) + 6$ backward analysis
- **Lemma** $S(P) \leq C(P) + \frac{1}{2}E(P) - 5$, two steps less by Offsetlemma!
- Kombination gives Induction-Base!
Theorem: SmartDFS is $\frac{4}{3}$ competitive

- Narrow passages optimal, sequence of $P_i$ independently!
- Only cells and steps, no edges!!
- Induction in $P_i$ over split-cell number! $S(P_i) \leq \frac{4}{3} C(P_i) - 2$
- Induction base: Use special lemmata!
**Induction base:** \( S(P_i) \leq \frac{4}{3}C(P_i) - 2 \)

- \( P_i \) no split-cell means, no split-cell in Layer 1
- Apply case-sensitive Lemma: \( C(P) + \frac{1}{2}E(P) - 5 \)
- Apply structural Lemma: \( E(P) \leq \frac{2}{3}C(P) + 6 \)

\[
S(P_i) \leq C(P_i) + \frac{1}{2}E(P_i) - 5 \\
\leq C(P_i) + \frac{1}{2} \left( \frac{2}{3}C(P_i) + 6 \right) - 5 \\
= \frac{4}{3}C(P_i) - 2
\]
Induction step: \( S(P_i) \leq \frac{4}{3} C(P_i) - 2 \)

- Split-cell in first layer of \( P_i \), otherwise done: Two Cases
- Split by \( c \) adjacent to some \( c' \)
- Typ (I) (curr. layer not) or Typ (II) (curr. layer fully.) component
- Split into \( P' \) and \( P'' \) with Rectangle/Square \( Q \)
- Case (i): \( Q = c \), otherwise \( Q \) smallest rectangle around \( c,c' \)
Case (i): \( S(P_i) \leq \frac{4}{3} C(P_i) - 2 \)

- \( S(P_i) = S(P') + S(P'') \)  (Gate) \( C(P_i) = C(P') + C(P'') - 1 \)
- Induction: For \( P' \) and \( P'' \) (less split-cells)

\[
S(P_i) = S(P') + S(P'') \leq \frac{4}{3} C(P') - 2 + \frac{4}{3} C(P'') - 2 \\
\leq \frac{4}{3} C(P_i) + \frac{4}{3} - 4 < \frac{4}{3} C(P_i) - 2
\]
Case (ii),(iii): \( S(P_i) \leq \frac{4}{3} C(P_i) - 2 \)

- \(|Q| = 4\) but save 4 steps!
- \(P', P''\) separately (I.H.) but
- Path in \(P_i\) from \(c'\) to \(c\) or from \(c\) to \(c'\) done in \(P', P''\)
- Save at least \(4=|Q|\) steps, two cells considered twice!
Case (ii), (iii): \( S(P_i) \leq \frac{4}{3} C(P_i) - 2 \)

- At least \( 4 = |Q| \) steps less, two cells argument
- \( S(P_i) = S(P') + S(P'') - 4 \) and \( C(P_i) = C(P') + C(P'') - 4 \)
- Apply I.H. for \( P' \) and \( P'' \)

\[
S(P_i) = S(P') + S(P'') - 4 \leq \frac{4}{3} C(P') + \frac{4}{3} C(P'') - 8 \\
\leq \frac{4}{3} \left( C(P') + C(P'') - 4 \right) - \frac{8}{3} < \frac{4}{3} C(P_i) - 2
\]
Summary SmartDFS

- Gridpolygons without holes: $\frac{7}{6}$
- Lower bound: $\frac{4}{3}$
- SmartDFS: $\frac{4}{3}$
- More sophisticated approach: approx. $\frac{5}{4}$
- Lower bound: $\frac{20}{17}$
- Optimal Offline Solution?